Robot Dynamics and Control

Nov 2018

Outline

- Dynamics
  - Model Identification
- Control
  - Natural (passive) systems
  - Computed torque control (decoupling)
  - Force (Compliance) Control
  - New trends in robotics

Dynamics - review

- Dynamics equation of motion:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) + \boldsymbol{\tau}_f(\boldsymbol{q},\dot{\boldsymbol{q}}) = \boldsymbol{\tau} + \boldsymbol{\tau}_e$$

$\boldsymbol{M}(\boldsymbol{q})$: Symmetric and Positive Definite:

- $\boldsymbol{M}(\boldsymbol{q}) = \boldsymbol{M}^T(\boldsymbol{q})$
- $\boldsymbol{q}^T\boldsymbol{M}(\boldsymbol{q})\boldsymbol{q} > 0, \ \forall \boldsymbol{q} \neq \boldsymbol{0}$
- $\dot{\boldsymbol{M}} - \boldsymbol{2C}$: skew symmetric

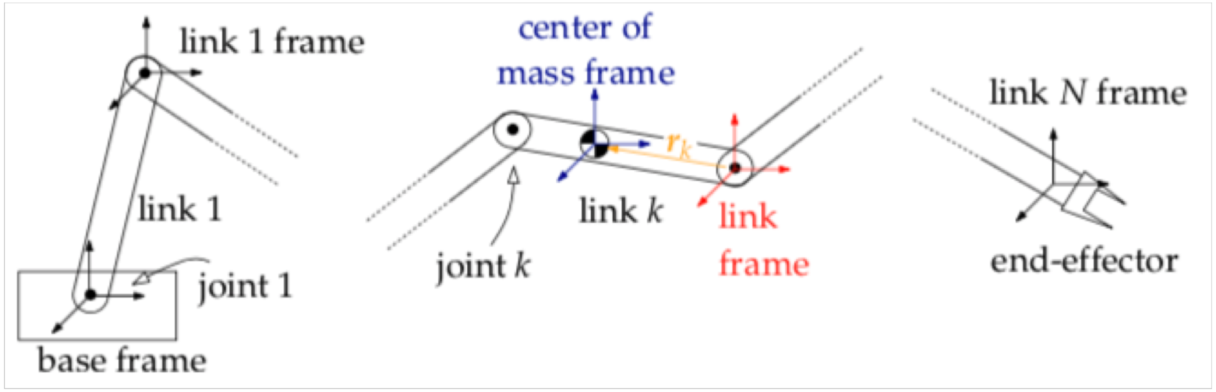
$\boldsymbol{\tau}_f(\boldsymbol{q},\dot{\boldsymbol{q}}) = \boldsymbol{F}_v\dot{\boldsymbol{q}} + \boldsymbol{F}_c\,\mathrm{sgn}(\dot{\boldsymbol{q}})$:Friction model (viscous + columb)

$\boldsymbol{\tau}_e = \boldsymbol{J}^T\boldsymbol{F}_e$:External force:

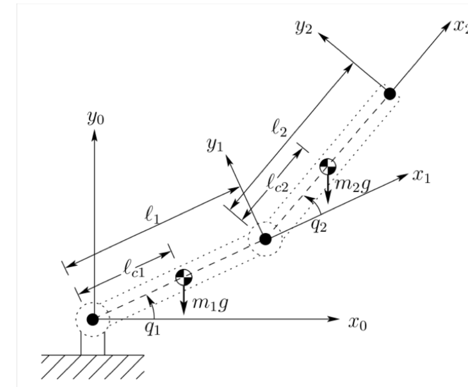
Dynamic Model Identification

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) + \boldsymbol{\tau}_f(\boldsymbol{q},\dot{\boldsymbol{q}}) = \boldsymbol{\tau} + \boldsymbol{\tau}_e$$

- How to get $\boldsymbol{M}(\boldsymbol{q})$ for a robot ? (aka Dynamic Model Identification)
  - $\boldsymbol{M}(\boldsymbol{q})$ requires knowledge of mass (m_k), Centre Of Mass ($\boldsymbol{r}_k$), inertia about COM ($\boldsymbol{I}_k$) of each link.



Dynamic Model Regression



- Dynamics are linear in dynamic parameters

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) + \boldsymbol{\tau}_f(\boldsymbol{q},\dot{\boldsymbol{q}}) \coloneqq \boldsymbol{Y}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\boldsymbol{\Theta} = \boldsymbol{\tau}$$

  - Each link gives 10 inertial parameters:
    - 1:mass (m_k),
    - 3:Centre Of Mass ($\boldsymbol{r}_k$),
    - 6:inertia about COM ($\boldsymbol{I}_k$) for each link k.
  - Friction force add 2 more parameters.
  - Gravity terms

$$\begin{bmatrix} m_1\ell_{c1}^2 + m_2(\ell_1^2 + \ell_{c2}^2 + 2\ell_1\ell_{c2}^2 + 2\ell_1\ell_{c2}\cos q_2) + I_1 + I_2 & * \\ m_2(\ell_{c2}^2 + \ell_1\ell_{c2}\cos q_2) + I_2 & m_2\ell_{c2}^2 + I_2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} h\dot{q}_2 & h\dot{q}_2 + h\dot{q}_1 \\ -h\dot{q}_1 & 0 \end{bmatrix} + \begin{bmatrix} (m_1\ell_{c1} + m_2\ell_1)g\cos q_1 + m_2\ell_{c2}g\cos(q_1+q_2) \\ m_2\ell_{c2}\cos(q_1+q_2) \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

$$h = -m_2\ell_1\ell_{c2}\sin q_2$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \ddot{q}_1 & \cos(q_2)(2\ddot{q}_1 + \ddot{q}_2) + \sin(q_2)(\dot{q}_1^2 - 2\dot{q}_1\dot{q}_2) & \ddot{q}_2 & g\cos(q_1) & g\cos(q_1+q_2) \\ 0 & \cos(q_2)\ddot{q}_1 + \sin(q_2)\dot{q}_1^2 & \ddot{q}_2 & 0 & g\cos(q_1+q_2) \end{bmatrix}}_{Y(q,\dot{q},\ddot{q})} \begin{bmatrix} m_1\ell_{c1}^2 + m_2(\ell_1^2 + \ell_{c2}^2) + I_1 + I_2 \\ m_2\ell_1\ell_{c2} \\ m_2\ell_1\ell_{c2} \\ m_1\ell_{c1} + m_2\ell_1 \\ m_2\ell_2 \end{bmatrix} \Rightarrow \Theta = \begin{bmatrix} \Theta_1 \\ \Theta_2 \\ \Theta_3 \\ \Theta_4 \\ \Theta_5 \end{bmatrix}$$

Dynamic Model Regression

Model identification:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) + \boldsymbol{\tau}_f(\boldsymbol{q}, \dot{\boldsymbol{q}}) \coloneqq \boldsymbol{Y}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})\boldsymbol{\Theta} = \boldsymbol{\tau}$$

- Each link gives 10 inertial parameters:
  - 1:mass (m_k),
  - 3:Centre Of Mass ($\boldsymbol{r}_k$),
  - 6:inertia about COM ($\boldsymbol{I}_k$) for each link k.

Identification Methods:

- **Computer-aided design (CAD) e.g. Solid works**

  not accurate
- **Dynamic model regression methods**

  common practice in industry

Dynamic Model Regression

- Move robot along trajectories and measure joint motions $\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}$ and torques $\boldsymbol{\tau}$

$$Y(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})\boldsymbol{\Theta} = \boldsymbol{\tau}$$

$$Z^N = \{q(t_i), \dot{q}(t_i), \ddot{q}(t_i), \tau(t_i),\ i = 1, 2, \ldots, N\}:$$

$$\underbrace{\begin{bmatrix} Y\ (q(t_1), \dot{q}(t_1), \ddot{q}(t_1)) \\ Y\ (q(t_2), \dot{q}(t_2), \ddot{q}(t_2)) \\ \vdots \\ Y\ (q(t_N), \dot{q}(t_N), \ddot{q}(t_N)) \end{bmatrix}}_{\Phi} \boldsymbol{\Theta} = \underbrace{\begin{bmatrix} \tau(t_1) \\ \tau(t_2) \\ \vdots \\ \tau(t_N) \end{bmatrix}}_{X}$$

- **Problem**: $\boldsymbol{\Theta}$ might not be identifiable because a number of parameters are zero or linearly dependent.
- Find $\boldsymbol{\Theta}_b$: minimum # of parameters to characterize dynamics
  - $\boldsymbol{\Phi}_b \boldsymbol{\Theta}_b = \boldsymbol{X}$, use SVD or QR factorization of $\boldsymbol{\Phi}$ to find $\boldsymbol{\Theta}_b$
- Procedure:
  - Robot is moved along a trajectory and joint motions $\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}$ and torques $\boldsymbol{\tau}$ are measured
  - Use weighted least square method to find $\boldsymbol{\Theta}_b$ that minimize $||\boldsymbol{\Phi}_b \boldsymbol{\Theta}_b - \boldsymbol{X}||_W$

$$\widehat{\boldsymbol{\Theta}}_b := \left(\boldsymbol{\Phi}_b^T W \boldsymbol{\Phi}_b\right)^{-1} \boldsymbol{\Phi}_b^T\, W\, \boldsymbol{X}$$

Dynamic Model Regression

- Procedure:
  - Robot is moved along a trajectory and joint motions $\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}$ and torques $\boldsymbol{\tau}$ are measured
  - Use weighted least square method to find $\boldsymbol{\Theta}_b$ that minimize $||\boldsymbol{\Phi}_b \boldsymbol{\Theta}_b - \boldsymbol{X}||_W$

$$\widehat{\boldsymbol{\Theta}}_{\boldsymbol{b}} := \left(\boldsymbol{\Phi}_b^T W \boldsymbol{\Phi}_{\boldsymbol{b}}\right)^{-\mathbf{1}} \boldsymbol{\Phi}_b^T\, W\, \boldsymbol{X}$$

  - To get good result, experiment design is important
  - Trajectory/path as a sum of sinusoid and covers all joint motions

  - Videos: Handbook of Robotics

[Dynamic identification of Kuka KR270: trajectory with load]

[Dynamic identification of Kuka LWR : Trajectory without load]

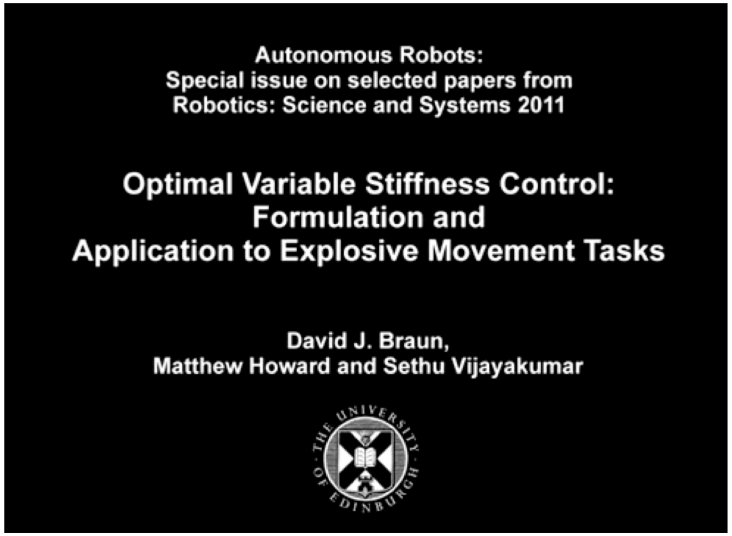
[Dynamic identification of Staubli TX40 : Trajectory with load]

Control

- Dynamics equation of motion:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) + \boldsymbol{\tau}_f(\boldsymbol{q},\dot{\boldsymbol{q}}) = \boldsymbol{\tau} + \boldsymbol{\tau}_e$$

- How to choose **τ** s.t. robot do what we want?



With **optimal** choice of **τ** : d = 5 m

Without **optimal** **τ** : d = 4 m

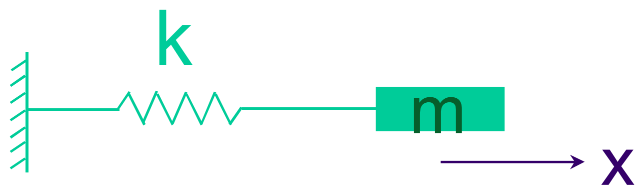
Natural (passive) systems

- **Review of simple 2nd order system:**
  - **Lagrange formulation**

$$\frac{d}{dt}\left(\frac{\partial(K-V)}{\partial\dot{x}}\right)-\frac{\partial(K-V)}{\partial x}=0$$

$$K=\frac{1}{2}m\dot{x}^2$$

$$V=\frac{1}{2}k\,x^2$$



$$\frac{d}{dt}\left(\frac{\partial K}{\partial\dot{x}}\right)-\frac{\partial K}{\partial x}=-\frac{\partial V}{\partial x}$$

$$m\ddot{x} = F = -kx$$

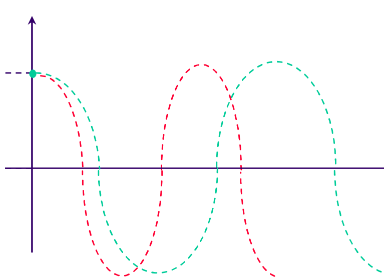
$$m\ddot{x} + kx = 0$$

Frequency increases
with stiffness
and inverse mass

Natural Frequency $\omega_n = \sqrt{\frac{k}{m}}$

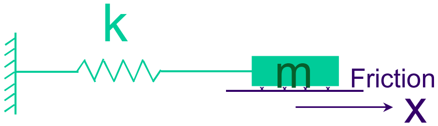
$$\ddot{x}+\omega_n^2 x = 0$$

$$x(t) = c\ \cos(\omega_n t+\phi)$$



- Dissipative systems

Dissipative Systems



$$\frac{d}{dt}\left(\frac{\partial(K-V)}{\partial \dot{x}}\right)-\frac{\partial(K-V)}{\partial x}=f_{friction}$$

Viscous friction: $f_{friction} = -b\dot{x}$

$$m\ddot{x} + b\dot{x} + kx = 0$$

$$\ddot{x}+\frac{b}{m}\dot{x}+\frac{k}{m}x=0$$

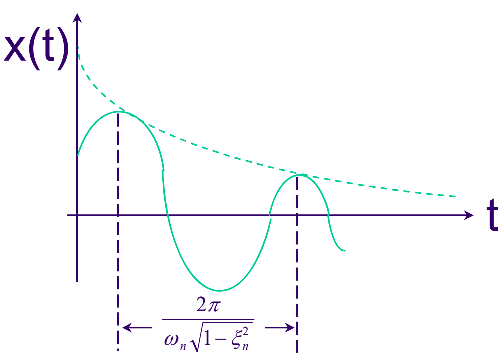
2nd order system

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\ddot{x} + 2\xi_n \omega_n \dot{x} + \omega_n^2 x = 0$$

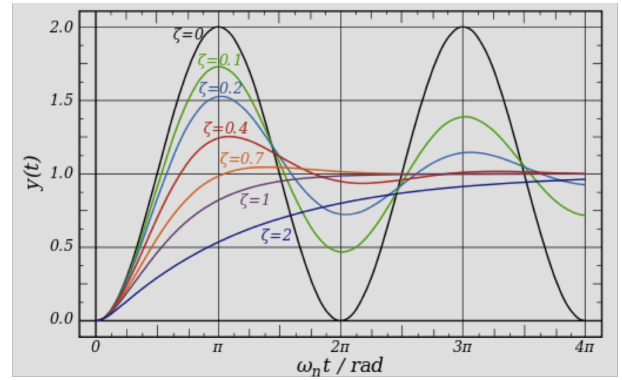
Natural frequency $\omega_n = \sqrt{\dfrac{k}{m}}$; $\xi_n = \dfrac{b}{2\sqrt{km}}$ Natural damping ratio

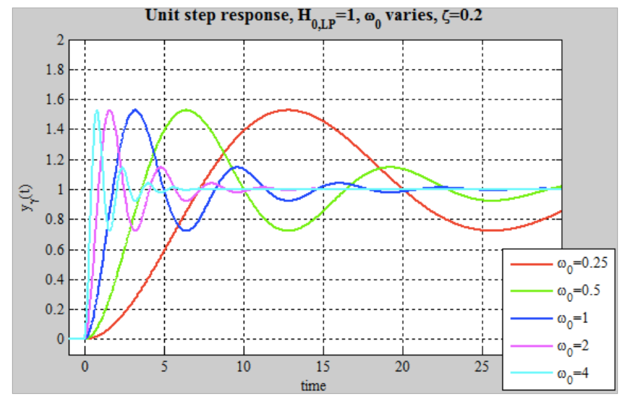
$$x(t) = ce^{-\xi_n \omega_n t}\cos(\underbrace{\omega_n\sqrt{1-\xi_n^2}}_{\omega}\,t + \phi)$$



damped Natural frequency

$$\omega = \omega_n\sqrt{1-\xi_n^2}$$





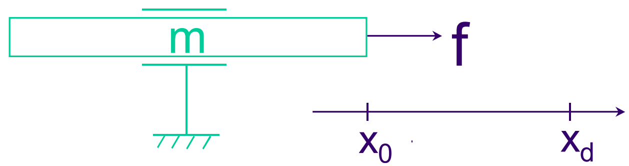
Higher **ω** : faster response

Higher **ζ**: slower response

ζ = 1 : optimal choice

Use **Control** to make robots behave like a **dissipative** system

1-dof Robot Control



$$m\ddot{x} = f$$

PD – control

Proportional + Derivative

$$m\ddot{x} = f = -k_p(x - x_d) - k_v\dot{x}$$

$$m\ddot{x} + k_v\dot{x} + k_p(x - x_d) = 0$$

Velocity gain   Position gain

$$1.\ \ddot{x} + \frac{k_v}{m}\dot{x} + \frac{k_p}{m}(x - x_d) = 0$$

$$1.\ \ddot{x} + 2\xi\omega\dot{x} + \omega^2(x - x_d) = 0$$

$$\xi = \frac{k_v}{2\sqrt{k_p m}}$$ closed loop damping ratio

$$\omega = \sqrt{\frac{k_p}{m}}$$ closed loop frequency

How to deal with nonlinearities?

Non Linearities

$$m\ddot{x} + b(x, \dot{x}) = f$$

Control Partitioning

$$f = \alpha f' + \beta$$

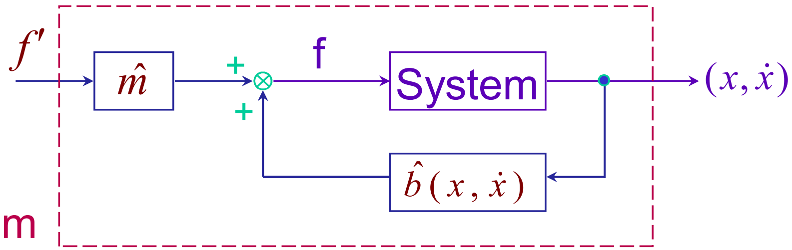
with

$$\alpha = \hat{m}$$

$$\beta = \hat{b}(x, \dot{x})$$

$$m\ddot{x} + b(x, \dot{x}) = \hat{m} f' + \hat{b}(x, \dot{x})$$

$$\longrightarrow \quad 1.\ddot{x} = f'$$



Unit mass system

Trajectory Tracking

$$x_d(t);\ \dot{x}_d(t);\ \text{and } \ddot{x}_d(t)$$

Control: $f' = \ddot{x}_d - k'_v(\dot{x} - \dot{x}_d) - k'_p(x - x_d)$

Closed-loop System:

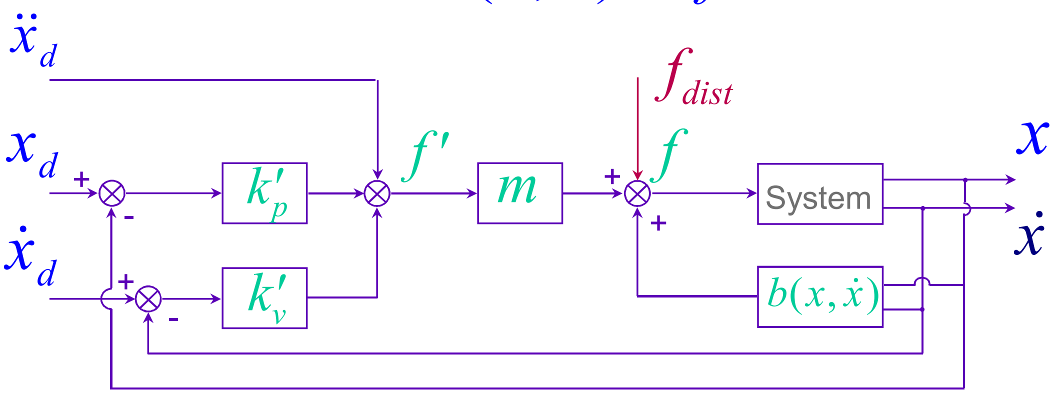
$$(\ddot{x} - \ddot{x}_d) + k'_v(\dot{x} - \dot{x}_d) + k'_p(x - x_d) = 0$$

with $e \equiv x - x_d$

$$\ddot{e} + k'_v\dot{e} + k'_p e = 0$$

Disturbance Rejection

$$m\ddot{x} + b(x,\dot{x}) = f$$



$$m\ddot{x} + b(x,\dot{x}) = f + f_{dist}$$

Control $f = mf' + b(x,\dot{x})$

Closed loop $\boxed{\ddot{e} + k'_v\dot{e} + k'_p e = \dfrac{f_{dist}}{m}}$

Steady-State Error

$$\ddot{e} + k_v'\dot{e} + k_p'e = \frac{f_{dist}}{m}$$

The steady-state $\left(\dot{e} = \ddot{e} = 0\right)$:

$$k_p'e = \frac{f_{dist}}{m}$$

$$e = \frac{f_{dist}}{mk_p'} = \frac{f_{dist}}{k_p}$$

Closed loop position gain (stiffness)

General Case:

Nonlinear Dynamic Decoupling

$$M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta) = \tau$$

$$\tau = \hat{M}(\theta)\underline{\underline{\tau'}} + \hat{V}(\theta,\dot{\theta}) + \hat{G}(\theta)$$

$$\mathbf{1}.\ddot{\theta} = (M^{-1}\hat{M})\tau' + M^{-1}[(V - \hat{V}) + (G - \hat{G})]$$

with perfect estimates

$$\mathbf{1}.\ddot{\theta} = \tau' + \varepsilon(t)$$

τ' : input of the unit-mass systems

$$\tau' = \ddot{\theta}_d - k_v'(\dot{\theta} - \dot{\theta}_d) - k_p'(\theta - \theta_d)$$

Closed-loop

$$\ddot{E} + k_v'\dot{E} + k_p'E = 0 + \varepsilon(t)$$

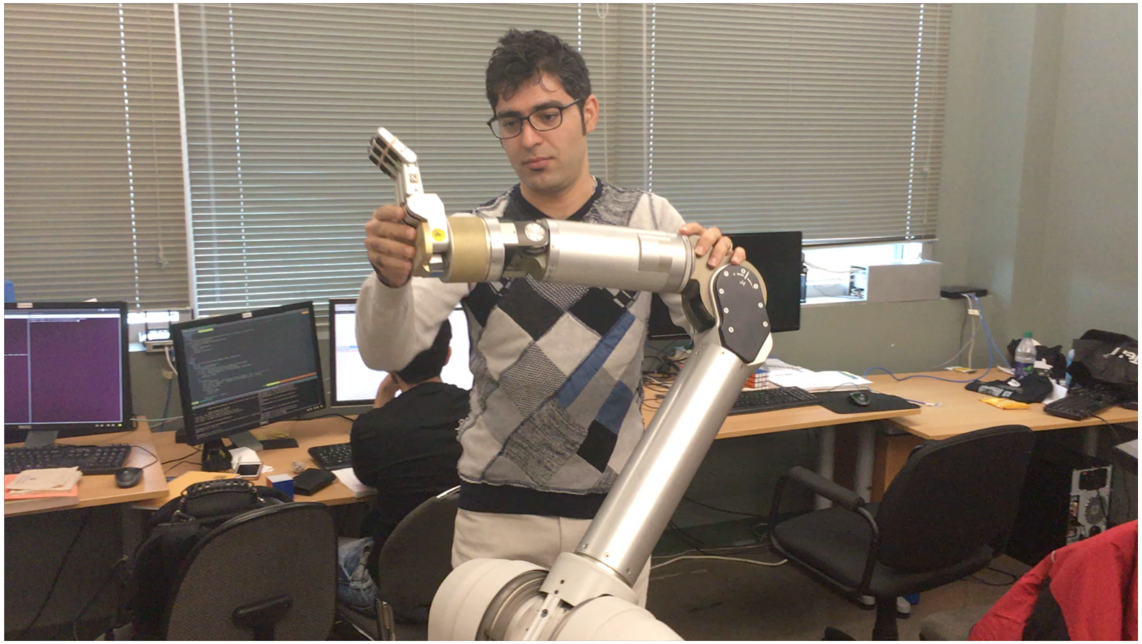
Rule of thumb:

V ≈ 0 usually don't estimate

- time consuming to compute

M , G: good estimate

- Gravity Compensation : $\boldsymbol{\tau} = \hat{\mathbf{G}}\ (\boldsymbol{\theta})$
- -> the robot is free in the air (**150K $** robot)



Practical issues for choosing Controller Gains

Performance

High Gains $\longrightarrow$ better disturbance rejection

Gains are limited by

- structural flexibilities
- time delays (actuator-sensing)
- sampling rate

$$\omega_n \leq \frac{\omega_{res}}{2} \quad \longleftarrow \text{lowest structural flexibility}$$

$$\omega_n \leq \frac{\omega_{delay}}{3} \quad \longleftarrow \text{largest delay} \left(\frac{2\pi}{\tau_{delay}}\right)$$

$$\omega_n \leq \frac{\omega_{sampling-rate}}{5}$$

Tacoma Narrows Bridge (1940)

Rule of thumb:

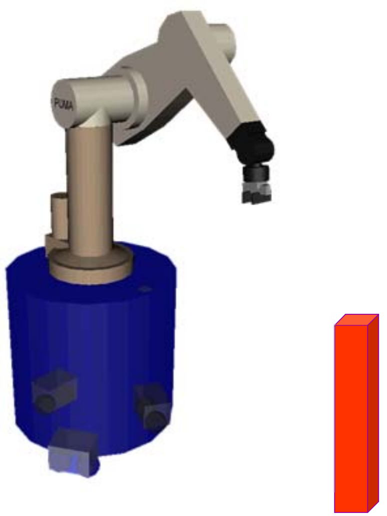
ω = (3 to 10 Hz)*2π

ζ = 1

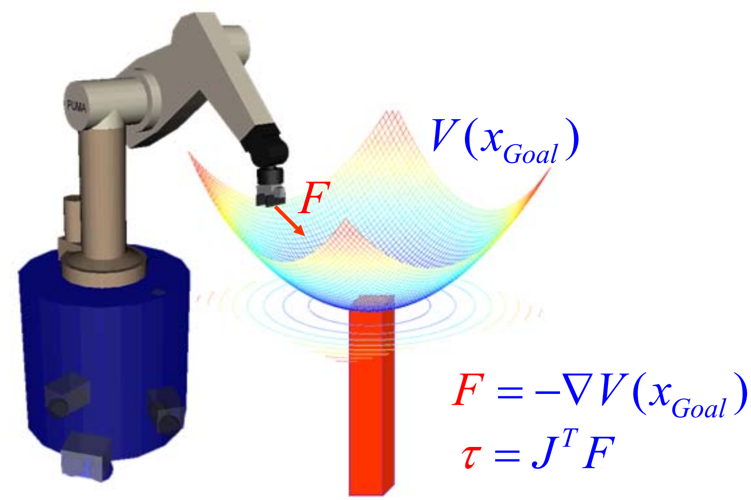
Task Oriented Control (Operational Space Control)

- Human does not do Joint Space Control
- Task space control is more intuitive

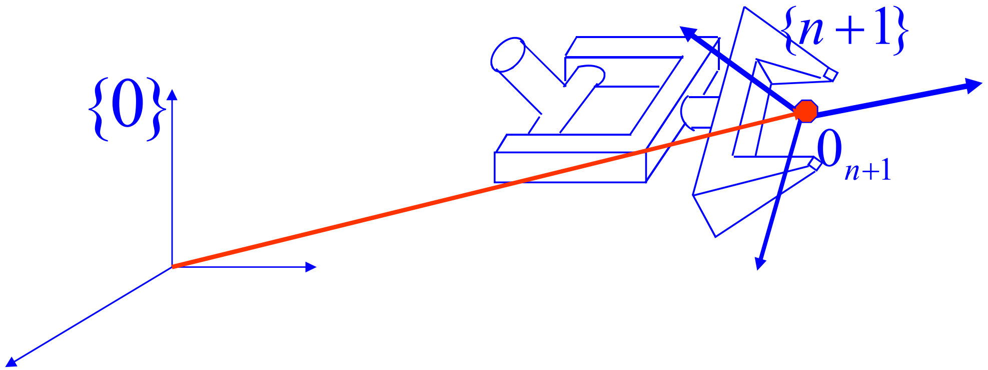
Joint Space Control



Operational Space Control



Task-Oriented Equations of Motion



Non-Redundant Manipulator ; $n = m$

$$x = \left(x_1\, x_2 \ldots x_m\right)^T$$

$$q = \left(q_1\, q_2 \ldots q_n\right)^T$$

Equations of Motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = F$$

$$x = \begin{pmatrix} x \\ y \\ z \\ \alpha \\ \beta \\ \gamma \end{pmatrix}$$

with

$$L(x, \dot{x}) = K(x, \dot{x}) - U(x)$$

Operational Space Dynamics

$$M_x(x)\ddot{x} + V_x(x,\dot{x}) + G_x(x) = F$$

$x:$	End-Effector Position and Orientation
$M_x(x):$	End-Effector Kinetic Energy Matrix
$V_x(x,\dot{x}):$	End-Effector Centrifugal and Coriolis forces
$G_x(x):$	End-Effector Gravity forces
$F:$	End-Effector Generalized forces

Joint Space/Task Space Relationships

Kinetic Energy

$$K_x(x,\dot{x}) \equiv K_q(q,\dot{q})$$

$$\frac{1}{2}\dot{x}^T M_x(x)\dot{x} \equiv \frac{1}{2}\dot{q}^T M(q)\dot{q}$$

Using $\dot{x} = J(q)\dot{q}$

$$\boxed{\frac{1}{2}\dot{q}^T (J^T M_x J)\,\dot{q} \equiv \frac{1}{2}\dot{q}^T M\,\dot{q}}$$

Joint Space/Task Space Relationships

$$M_x(x) = J^{-T}(q)\,M(q)\,J^{-1}(q)$$

$$V_x(x,\dot{x}) = J^{-T}(q)\,V(q,\dot{q}) - M_x(q)\,h(q,\dot{q})$$

$$G_x(x) = J^{-T}(q)\,G(q)$$

where $h(q,\dot{q}) \doteq \dot{J}(q)\dot{q}$

Nonlinear Dynamic Decoupling

Model

$$M_x(x)\ddot{x} + V_x(x,\dot{x}) + G_x(x) = F$$

Control Structure

$$F = \hat{M}(x)F' + \hat{V}_x(x,\dot{x}) + \hat{G}_x(x)$$

Decoupled System

$$I\,\ddot{x} = F'$$

with $\tau = J^T F$

Trajectory Tracking

Trajectory: $x_d\,,\ \dot{x}_d\,,\ \ddot{x}_d$

$$F' = I\,\ddot{x}_d - k_v'(\dot{x} - \dot{x}_d) - k_p'(x - x_d)$$

$$(\ddot{x} - \ddot{x}_d) + k_v'(\dot{x} - \dot{x}_d) + k_p'(x - x_d) = 0$$

or $\boxed{\ddot{\varepsilon}_x + k_v'\dot{\varepsilon}_x + k_p'\varepsilon_x = 0}$

with $\varepsilon_x = x - x_d$

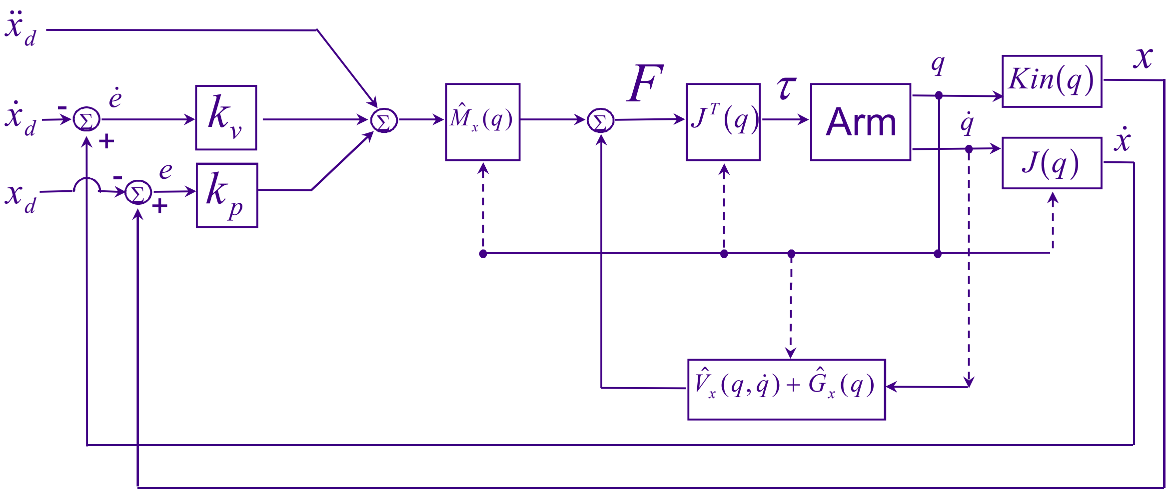
In joint space

$$\ddot{\varepsilon}_q + k_v'\dot{\varepsilon}_q + k_p'\varepsilon_q = 0$$

with $\varepsilon_q = q - q_d$

Overview of the Controller

Task-Oriented Control



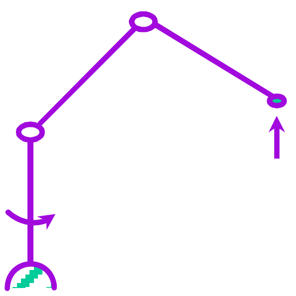
Compliance Control
Force Control

Compliance

$$\boxed{I\ \ddot{x} = F'}$$

$$F' = -\begin{pmatrix} k'_{p_x} & 0 & 0 \\ 0 & k'_{p_y} & 0 \\ 0 & 0 & k'_{p_z} \end{pmatrix}(x - x_d) - k'_v \dot{x}$$

set to zero



$$\ddot{x} + k_v^{'}\dot{x} + k_{px}^{'}(x - x_d) = 0$$

$$\ddot{y} + k_v^{'}\dot{y} + k_{py}^{'}(y - y_d) = 0$$

$$\ddot{z} + k_v^{'}\dot{z} = 0$$

Compliance along Z

Stiffness

$$\ddot{z} + k'_v \dot{z} + k'_{p_z}(z - z_d) = 0$$

determines stiffness along $\underline{z}$

Closed-Loop Stiffness: $\hat{M}_x k'_p = k_p$

$$F = K_x(x - x_d)$$

$$\tau = J^T F = J^T K_x \Delta x = (J^T K_x J)\Delta\theta = K_\theta \Delta\theta$$

$$\boxed{K_\theta = J^T(\theta) K_x J(\theta)}$$

Applications

- Compliance Control