CMPUT 412 Robotics: Control

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Outline

- Control
  - Natural (passive) systems
  - Computed torque control (decoupling)
  - Force (Compliance) Control
  - New trends in robotics

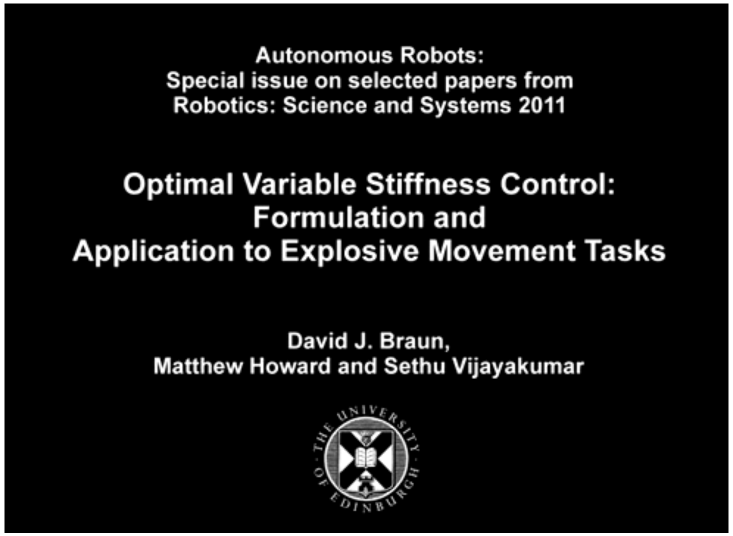
Control

- Dynamics equation of motion:

$$M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta) = \tau$$

- How to choose **τ** s.t. robot do what we want?

Video:
throwing a ball with
2-link robot
(using compliant
actuators)



With **optimal** choice
of **τ** : d = 5 m

Without **optimal** **τ** :
d = 4 m

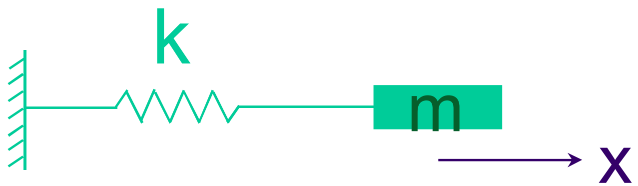
Natural (passive) systems

- **Simple 2nd order system:**
  - **Lagrange formulation**

$$\frac{d}{dt}\left(\frac{\partial(K-V)}{\partial \dot{x}}\right)-\frac{\partial(K-V)}{\partial x}=0$$

$$K=\frac{1}{2}m\dot{x}^2$$

$$V=\frac{1}{2}k\,x^2$$



$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right)-\frac{\partial K}{\partial x}=-\frac{\partial V}{\partial x}$$

$$m\ddot{x} = F = -kx$$

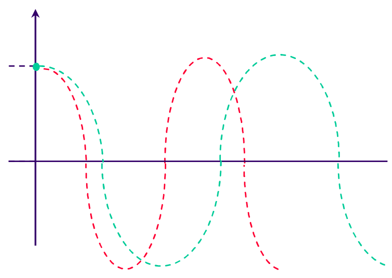
$$m\ddot{x} + kx = 0$$

Frequency increases
with stiffness
and inverse mass

Natural Frequency $\omega_n = \sqrt{\frac{k}{m}}$

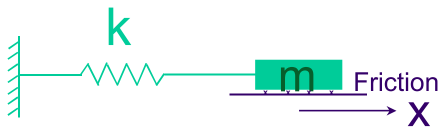
$$\ddot{x} + \omega_n^2 x = 0$$

$$x(t) = c\ \cos(\omega_n t + \phi)$$



- Dissipative systems

Dissipative Systems



$$\frac{d}{dt}\left(\frac{\partial(K-V)}{\partial\dot{x}}\right)-\frac{\partial(K-V)}{\partial x}=f_{friction}$$

Viscous friction: $f_{friction}=-b\dot{x}$

$$m\ddot{x}+b\dot{x}+kx=0$$

$$\ddot{x}+\frac{b}{m}\dot{x}+\frac{k}{m}x=0$$

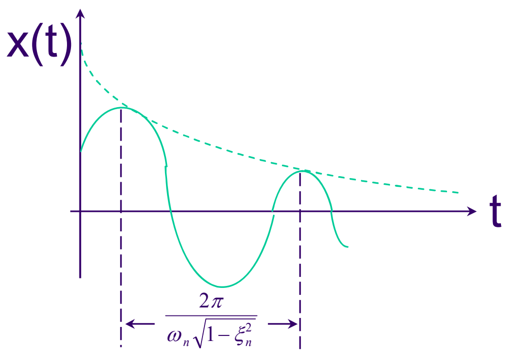
2^{nd} order system

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\ddot{x} + 2\xi_n \omega_n \dot{x} + \omega_n^2 x = 0$$

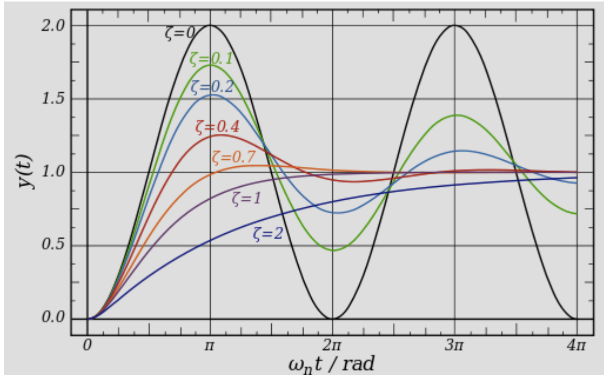
Natural frequency $\omega_n = \sqrt{\frac{k}{m}}$; $\xi_n = \frac{b}{2\sqrt{km}}$ Natural damping ratio

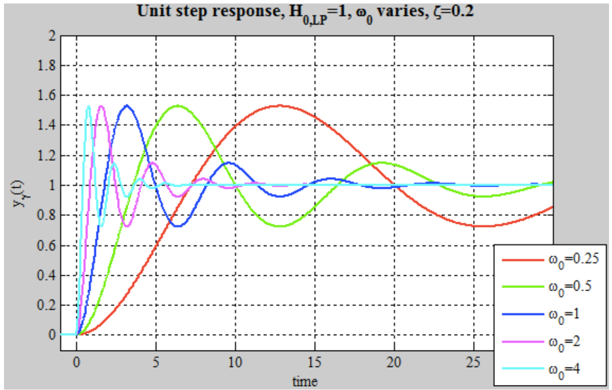
$$x(t) = ce^{-\xi_n \omega_n t}\cos(\underbrace{\omega_n \sqrt{1-\xi_n^2}}_{\omega}\, t + \phi)$$



damped Natural frequency

$$\omega = \omega_n \sqrt{1-\xi_n^2}$$





Higher **ω** : faster response

Higher **ζ**: slower response

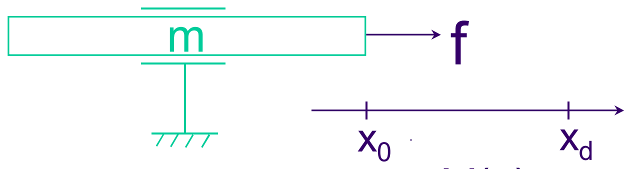
optimal choice (no overshoot) : **ζ = 1**

2^{nd} order systems are Practical

With PD-control, we make the 1^{st} joint of WAM robot behave like a 2^{nd} order system

Use **Control** to make robots behave like a **dissipative** system

1-dof Robot Control



$$m\ddot{x} = f$$

$$m\ddot{x} = f = -k_p(x - x_d) - k_v\dot{x}$$

PD – control

Proportional + Derivative

$$m\ddot{x} + k_v\dot{x} + k_p(x - x_d) = 0$$

Velocity gain (k_v) Position gain (k_p)

$$1.\ \ddot{x} + \frac{k_v}{m}\dot{x} + \frac{k_p}{m}(x - x_d) = 0$$

$$1.\ \ddot{x} + 2\xi\omega\dot{x} + \omega^2(x - x_d) = 0$$

$$\xi = \frac{k_v}{2\sqrt{k_p m}}$$ closed loop damping ratio

$$\omega = \sqrt{\frac{k_p}{m}}$$ closed loop frequency

How to deal with non-linearity?

Non Linearities

$$m\ddot{x} + b(x, \dot{x}) = f$$

Control Partitioning

$$f = \alpha f' + \beta$$

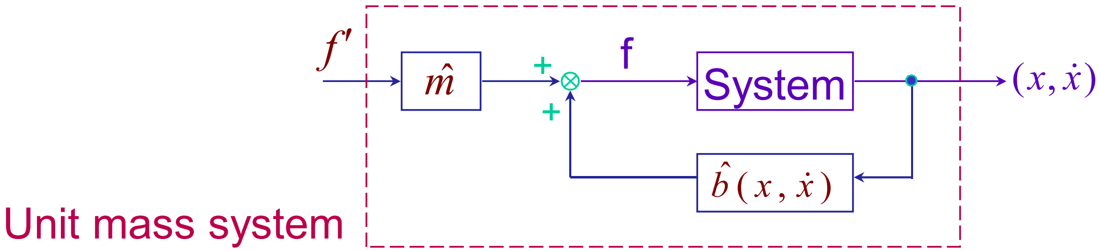
with

$$\alpha = \hat{m}$$

$$\beta = \hat{b}(x, \dot{x})$$

$$m\ddot{x} + b(x, \dot{x}) = \hat{m} f' + \hat{b}(x, \dot{x})$$

$$\longrightarrow \quad 1.\ddot{x} = f'$$



Trajectory Tracking

$$x_d(t);\ \dot{x}_d(t);\ \text{and}\ \ddot{x}_d(t)$$

Control: $f' = \ddot{x}_d - k'_v(\dot{x} - \dot{x}_d) - k'_p(x - x_d)$

Closed-loop System:

$$(\ddot{x} - \ddot{x}_d) + k'_v(\dot{x} - \dot{x}_d) + k'_p(x - x_d) = 0$$

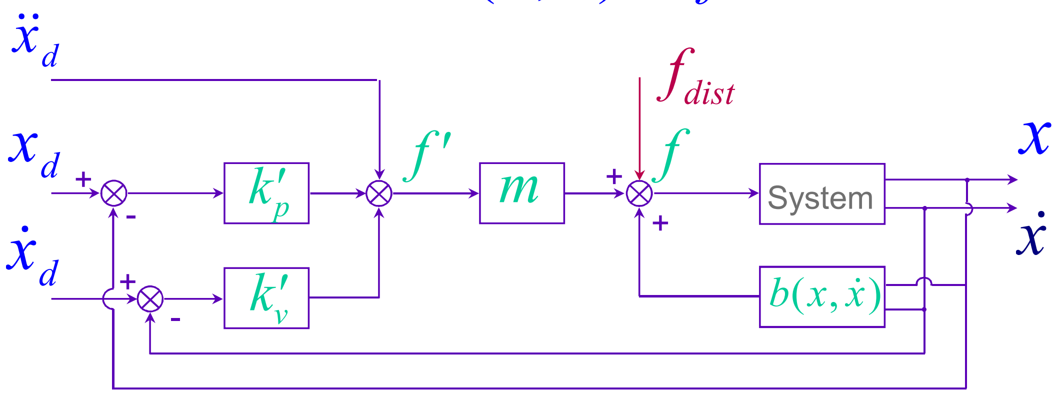
with $e \equiv x - x_d$

$$\ddot{e} + k'_v\dot{e} + k'_p e = 0$$

Effect of disturbance ?

Disturbance Rejection

$$m\ddot{x} + b(x,\dot{x}) = f$$



$$m\ddot{x} + b(x,\dot{x}) = f + f_{dist}$$

Control $f = mf' + b(x,\dot{x})$

Closed loop

$$\ddot{e} + k'_v\dot{e} + k'_p e = \frac{f_{dist}}{m}$$

Steady-State Error

$$\ddot{e} + k_v'\dot{e} + k_p'e = \frac{f_{dist}}{m}$$

The steady-state $\left(\dot{e} = \ddot{e} = 0\right)$:

$$k_p'e = \frac{f_{dist}}{m}$$

$$e = \frac{f_{dist}}{mk_p'} = \frac{f_{dist}}{k_p}$$

Closed loop position gain (stiffness)

General Case:

Nonlinear Dynamic Decoupling

$$M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta) = \tau$$

$$\tau = \hat{M}(\theta)\underline{\tau'} + \hat{V}(\theta,\dot{\theta}) + \hat{G}(\theta)$$

$$\mathbf{1}.\ddot{\theta} = (M^{-1}\hat{M})\tau' + M^{-1}[(V-\hat{V}) + (G-\hat{G})]$$

with perfect estimates

$$\mathbf{1}.\ddot{\theta} = \tau' + \varepsilon(t)$$

τ' : input of the unit-mass systems

$$\tau' = \ddot{\theta}_d - k'_v(\dot{\theta} - \dot{\theta}_d) - k'_p(\theta - \theta_d)$$

Closed-loop

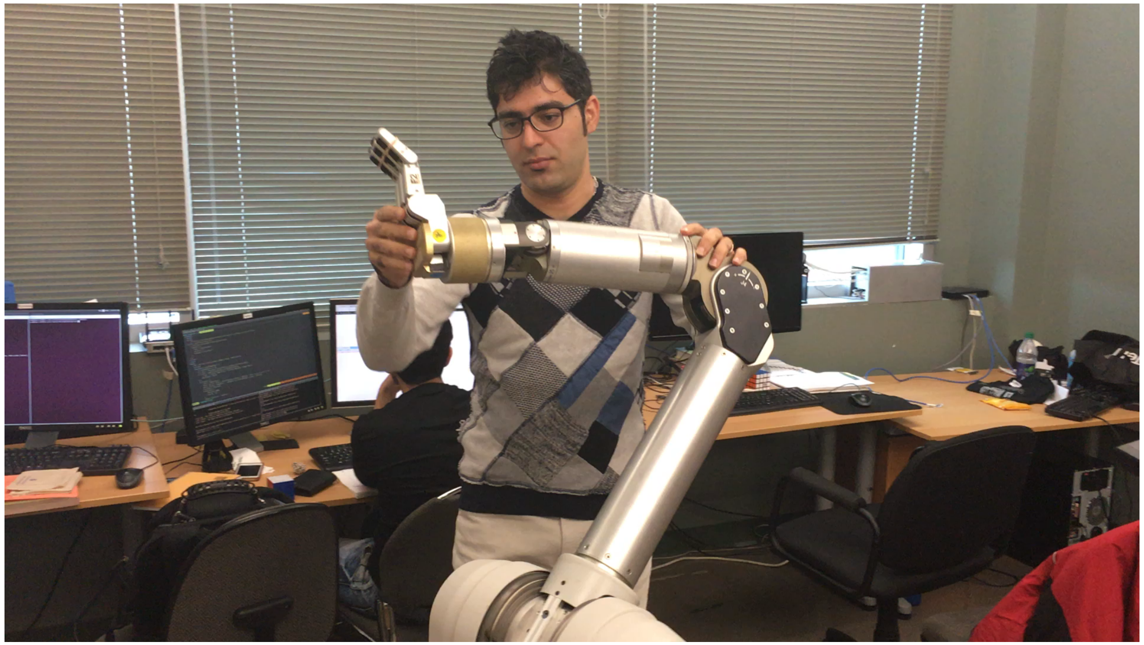
$$\ddot{E} + k'_v\dot{E} + k'_p E = 0 + \varepsilon(t)$$

Rule of thumb:

V ≈ 0 : usually we do not estimate V

M, G: good estimate

- Gravity Compensation : $\boldsymbol{\tau} = \hat{\mathbf{G}}(\boldsymbol{\theta})$
- -> the robot is free in the air



Practical issues for choosing Controller Gains

Performance

High Gains → better disturbance rejection

Gains are limited by

- structural flexibilities
- time delays (actuator-sensing)
- sampling rate

$$\omega_n \le \frac{\omega_{res}}{2}$$ ← lowest structural flexibility

$$\omega_n \le \frac{\omega_{delay}}{3}$$ ← largest delay $\left(\frac{2\pi}{\tau_{delay}}\right)$

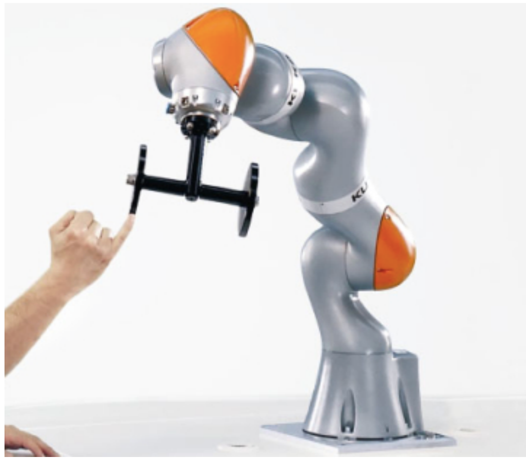
$$\omega_n \le \frac{\omega_{sampling-rate}}{5}$$

Tacoma Narrows Bridge (1940)

Rule of thumb:
ω = (3 to 10 Hz)*2π
ζ = 1

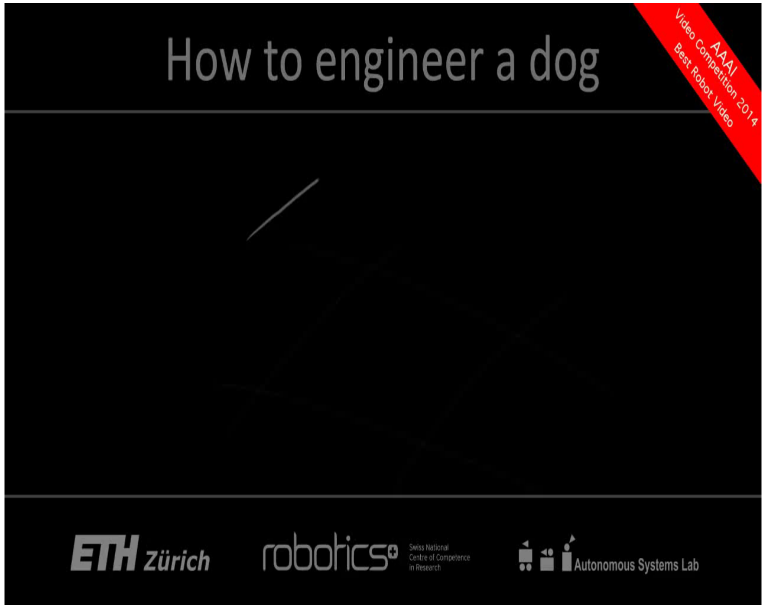
New Trends in Robotics (Force Control)

- Human-robot-interaction (HRI) ->
- Light-weight robots (force controlled)
- Examples are WAM, Kuka iiwa, …



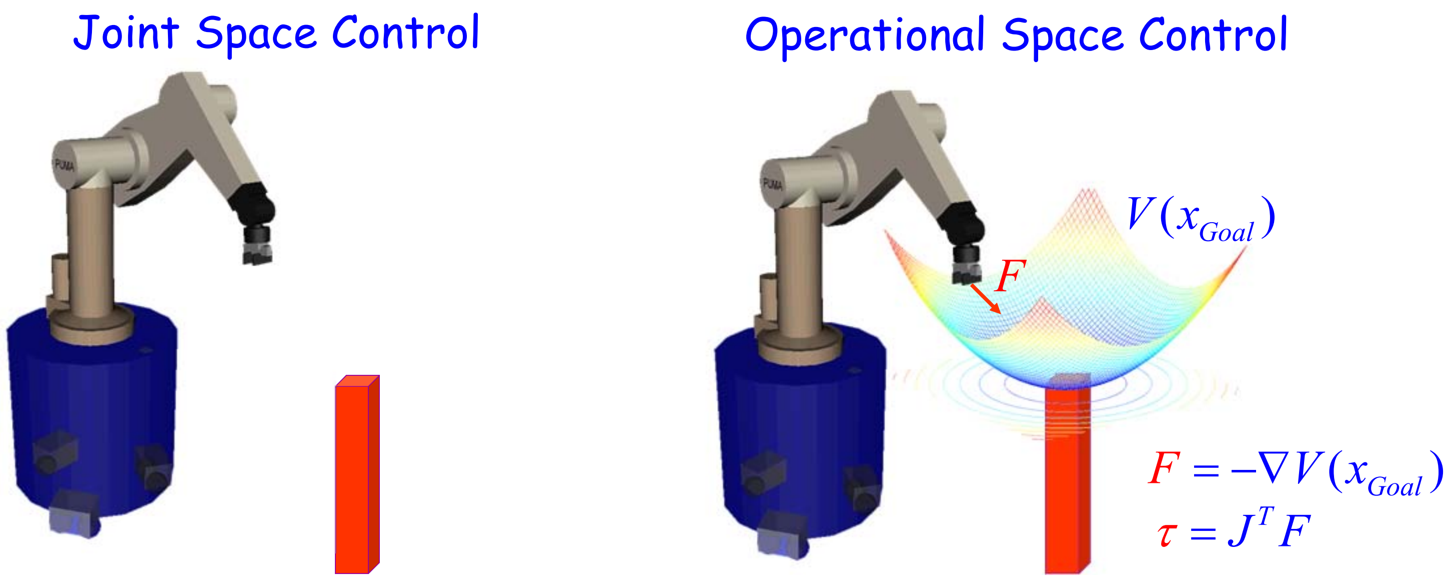
- even if you have torque control (closed-loop), we have to handle impact (open-loop)
- Variable Stiffness Actuators (VSA) ->
- Inherent (passive) compliance -> optimal control

ETH robot:
StarlETH



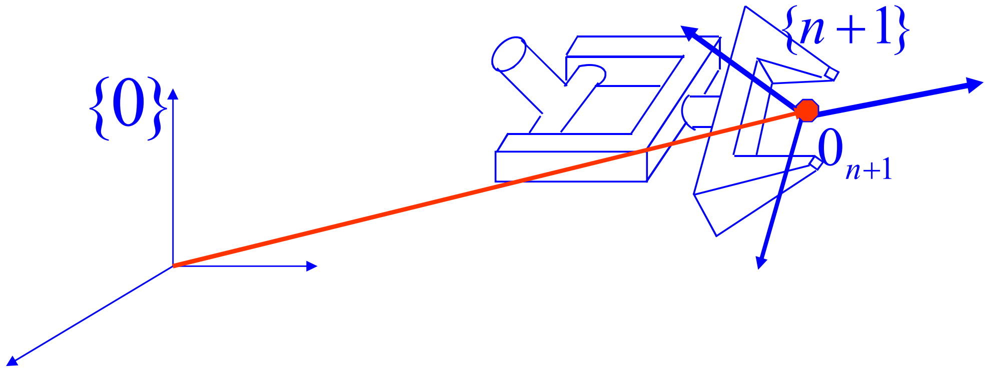
Task Oriented Control (Operational Space Control)

- Human do not use Joint Space Control
- Task space control is more intuitive



Taken from O. Khatib (Introduction to Robotics course)

Task-Oriented Equations of Motion



Non-Redundant Manipulator ; $n = m$

$$x = \left(x_1\, x_2 \ldots x_m\right)^T$$

$$q = \left(q_1\, q_2 \ldots q_n\right)^T$$

Equations of Motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = F$$

$$x = \begin{pmatrix} x \\ y \\ z \\ \alpha \\ \beta \\ \gamma \end{pmatrix}$$

with

$$L(x, \dot{x}) = K(x, \dot{x}) - U(x)$$

Operational Space Dynamics

$$M_x(x)\ddot{x} + V_x(x,\dot{x}) + G_x(x) = F$$

x :	End-Effector Position and Orientation
$M_x(x)$:	End-Effector Kinetic Energy Matrix
$V_x(x,\dot{x})$:	End-Effector Centrifugal and Coriolis forces
$G_x(x)$:	End-Effector Gravity forces
F :	End-Effector Generalized forces

Joint Space/Task Space Relationships

Kinetic Energy

$$K_x(x,\dot{x}) \equiv K_q(q,\dot{q})$$

$$\frac{1}{2}\dot{x}^T M_x(x)\dot{x} \equiv \frac{1}{2}\dot{q}^T M(q)\dot{q}$$

Using $\dot{x} = J(q)\dot{q}$

$$\boxed{\frac{1}{2}\dot{q}^T (J^T M_x J)\, \dot{q} \equiv \frac{1}{2}\dot{q}^T M\, \dot{q}}$$

Joint Space/Task Space Relationships

$$M_x(x) = J^{-T}(q)\,M(q)\,J^{-1}(q)$$

$$V_x(x,\dot{x}) = J^{-T}(q)\,V(q,\dot{q}) - M_x(q)\,h(q,\dot{q})$$

$$G_x(x) = J^{-T}(q)\,G(q)$$

where $h(q,\dot{q}) \doteq \dot{J}(q)\dot{q}$

Nonlinear Dynamic Decoupling

Model

$$M_x(x)\ddot{x} + V_x(x,\dot{x}) + G_x(x) = F$$

Control Structure

$$F = \hat{M}(x)F' + \hat{V}_x(x,\dot{x}) + \hat{G}_x(x)$$

Decoupled System

$$I\,\ddot{x} = F'$$

with $\tau = J^T F$

Trajectory Tracking

Trajectory: x_d, $\dot{x}_d$, $\ddot{x}_d$

$$F' = I\,\ddot{x}_d - k_v'(\dot{x} - \dot{x}_d) - k_p'(x - x_d)$$

$$(\ddot{x} - \ddot{x}_d) + k_v'(\dot{x} - \dot{x}_d) + k_p'(x - x_d) = 0$$

or $$\ddot{\varepsilon}_x + k_v'\dot{\varepsilon}_x + k_p'\varepsilon_x = 0$$

with $\varepsilon_x = x - x_d$

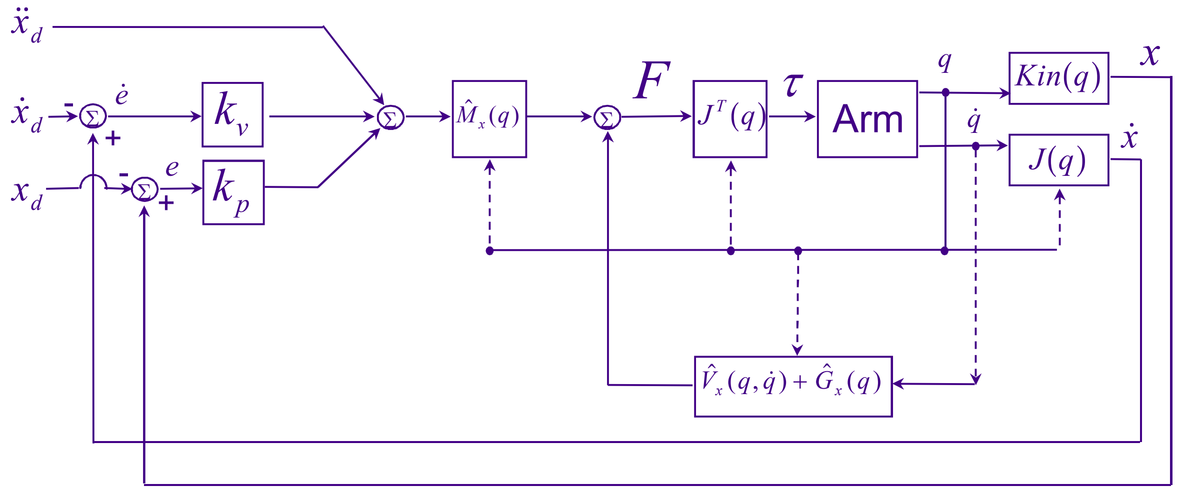
In joint space

$$\ddot{\varepsilon}_q + k_v'\dot{\varepsilon}_q + k_p'\varepsilon_q = 0$$

with $\varepsilon_q = q - q_d$

Overview of the Controller

Task-Oriented Control



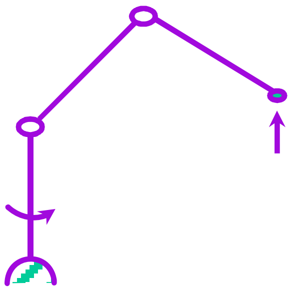
Compliance Control
Force Control

Compliance

$$I\ \ddot{x} = F'$$

$$F' = -\begin{pmatrix} k'_{p_x} & 0 & 0 \\ 0 & k'_{p_y} & 0 \\ 0 & 0 & k'_{p_z} \end{pmatrix}(x - x_d) - k'_v \dot{x}$$

set to zero



$$\ddot{x} + k'_v \dot{x} + k'_{px}(x - x_d) = 0$$

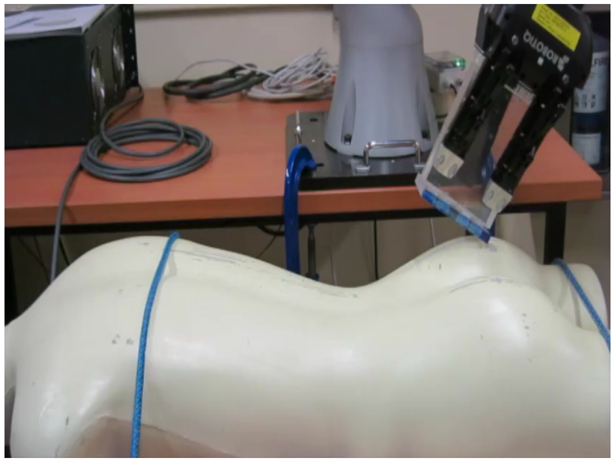
$$\ddot{y} + k'_v \dot{y} + k'_{py}(y - y_d) = 0$$

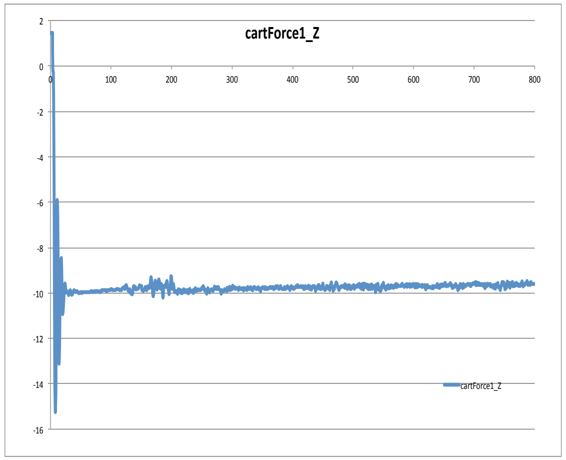
$$\ddot{z} + k'_v \dot{z} = 0$$

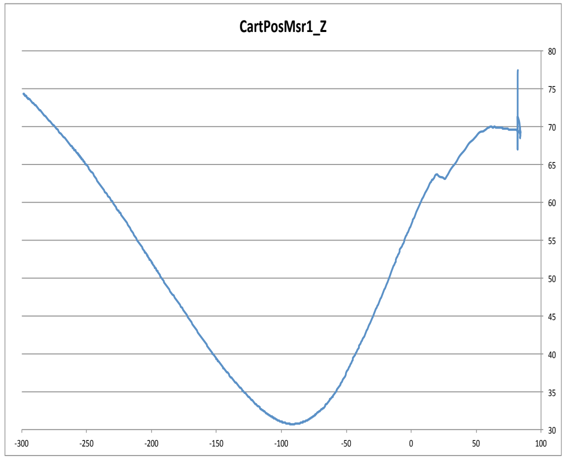
Compliance along Z

Unknown Surface following

- Compliance Controller
- Soft in Z (contact direction)
- Stiff in X, Y, and rotations







Compliance Control

- 1-DOF compliance control

https://www.youtube.com/watch?v=WS1gSRcJbJQ

Conversion of Compliance in Task-space and Joint-space

Stiffness

$$\ddot{z} + k'_v \dot{z} + k'_{p_z}(z - z_d) = 0$$

determines stiffness along z

Closed-Loop Stiffness: $\hat{M}_x k'_p = k_p$

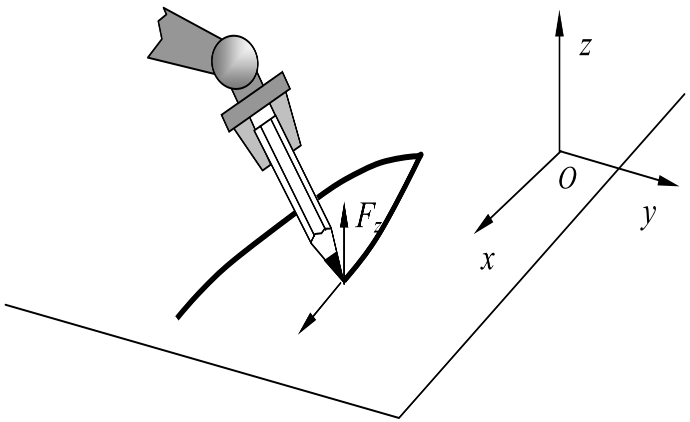
$$F = K_x(x - x_d)$$

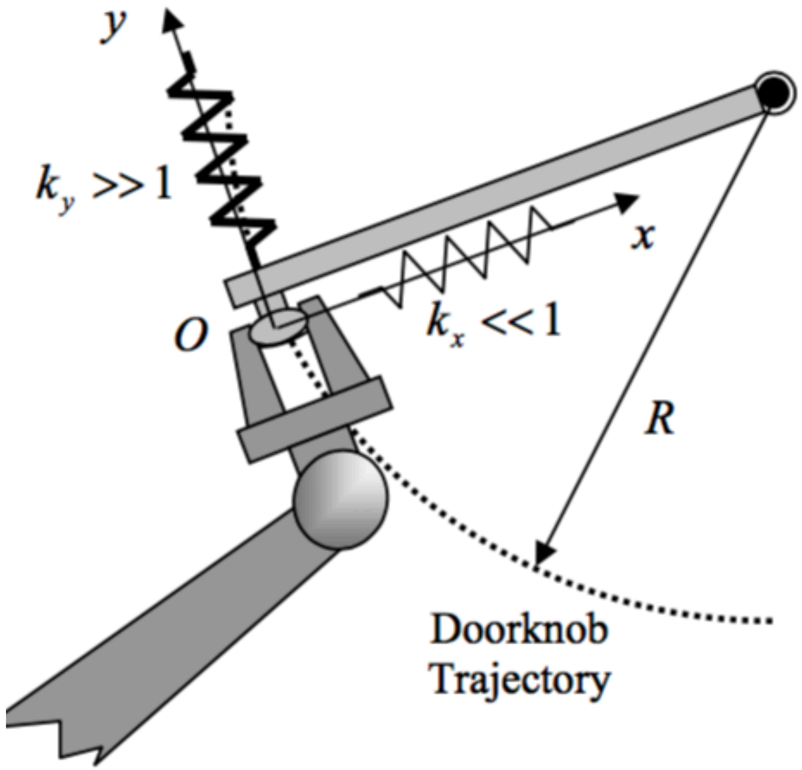
$$\tau = J^T F = J^T K_x \Delta x = (J^T K_x J)\Delta\theta = K_\theta \Delta\theta$$

$$\boxed{K_\theta = J^T(\theta) K_x J(\theta)}$$

Example Applications

- Compliance Control





Reinforcement Learning Based Control

Thank You !

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