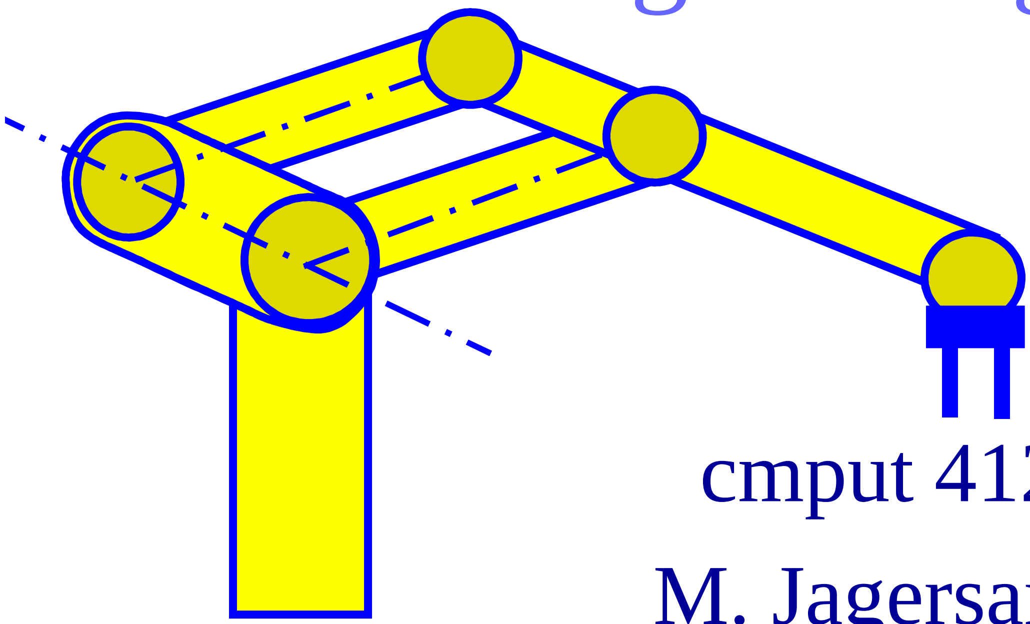
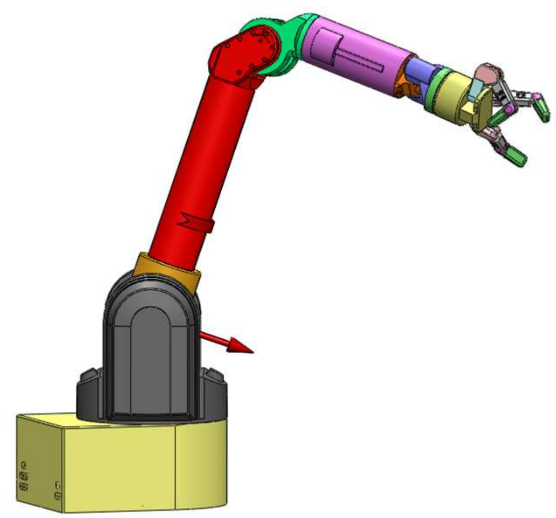
Robot Kinematics and linkage configurations 2



cmput 412

M. Jagersand

With slides from A. Shademan
and A Casals



Review Kinematics

Forward Kinematics (angles to position)

What you are given:	The length of each link The angle of each joint
What you can find:	The position of any point (i.e. it's (x, y, z) coordinates

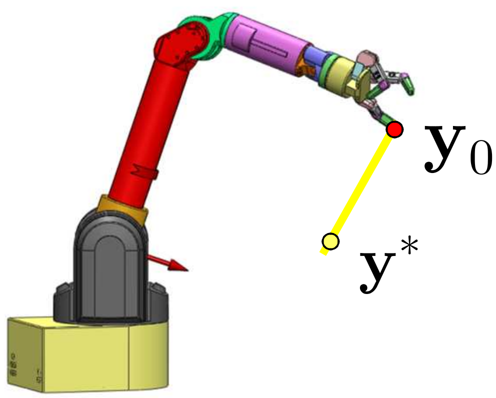
Inverse Kinematics (position to angles)

What you are given:	The length of each link The position of some point on the robot
What you can find:	The angles of each joint needed to obtain that position

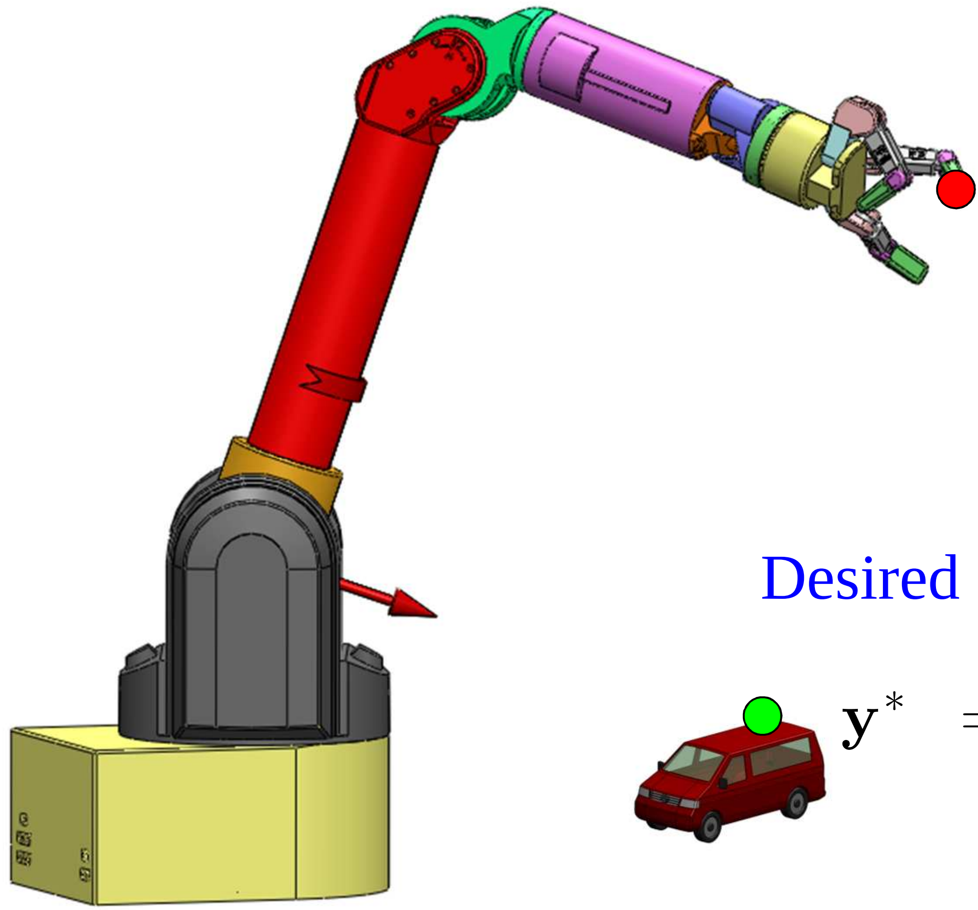
Numerical Inverse Kinematics

- Cartesian Location $\mathbf{y} = [x, y, z]^T \;=\; f(\mathrm{x})$
- Motor joint angles: $\mathbf{x} = [\, x_1, \quad x_2, \ldots \quad x_n \,]^T$
- Local linear model: $\Delta\mathbf{y} = \mathbf{J}(\mathbf{x})\Delta\mathbf{x}$
- Numerical steps: 1 Solve: $\mathbf{y}^* - \mathbf{y}_k = \mathbf{J}\Delta\mathbf{x}$

  2 Update: $\mathbf{x}_{k+1} = \mathbf{x}_k + \Delta\mathbf{x}$



Cartesian Trajectory Motion



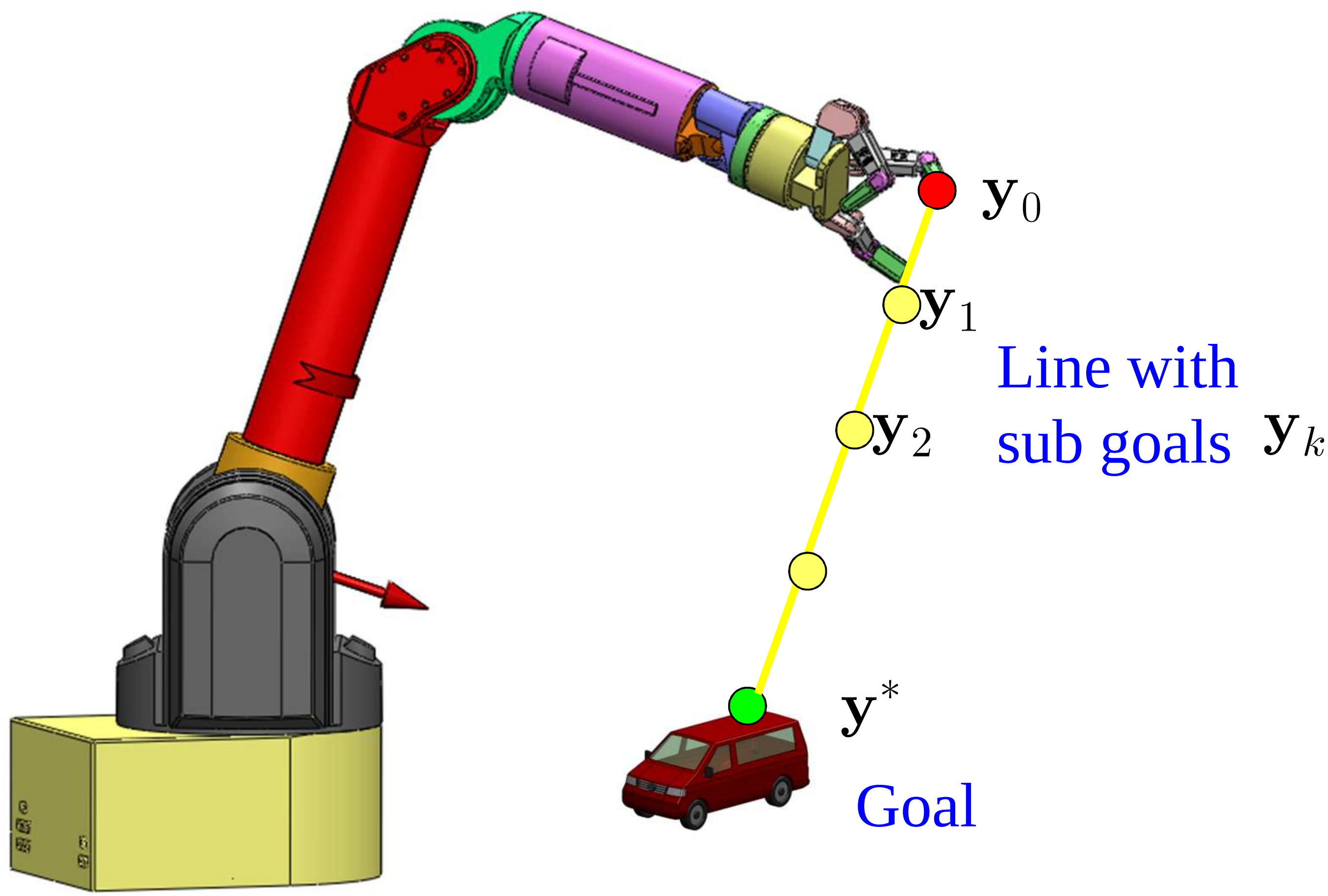
Current
Cartesian
position

$\mathbf{y}_0$

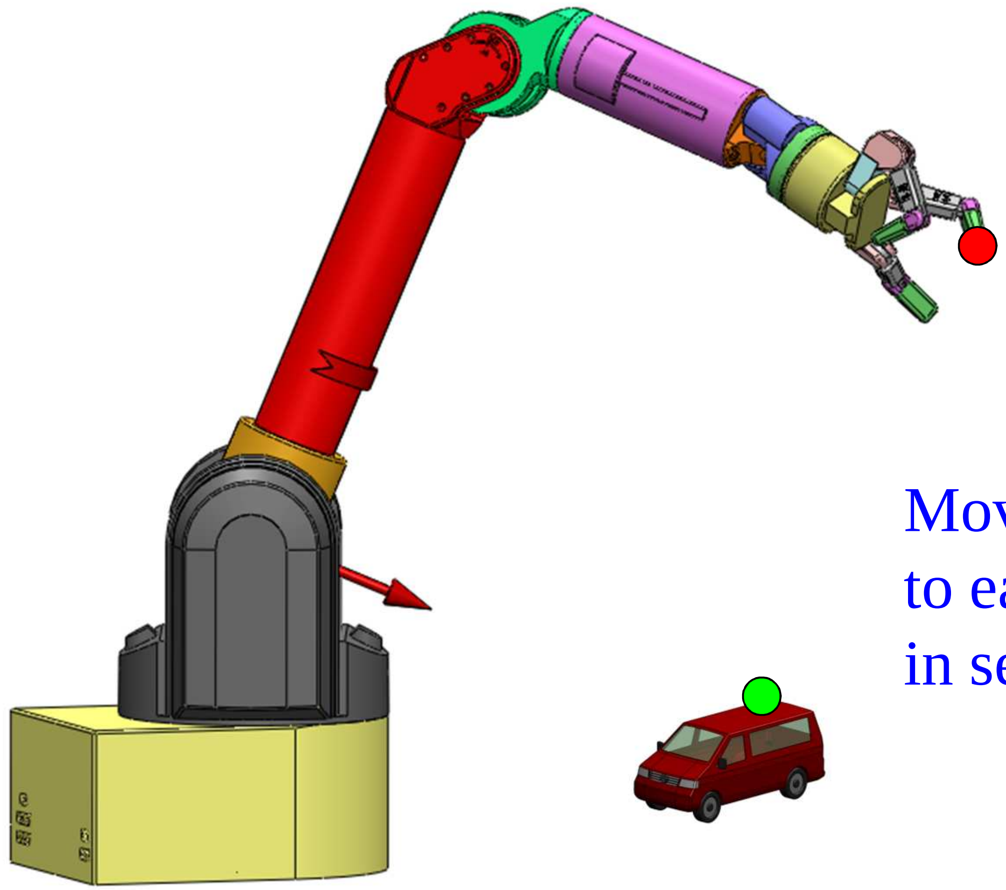
Desired goal:

$$\mathbf{y}^* = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Cartesian Trajectory Motion

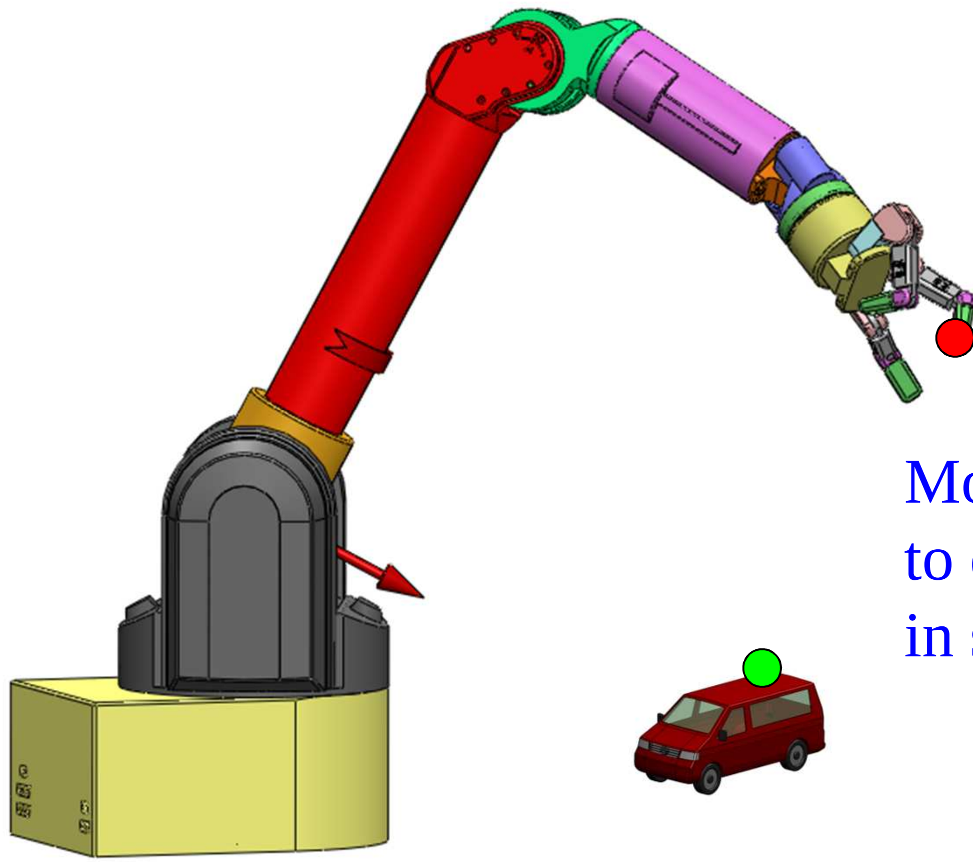


Cartesian Trajectory Motion



Move the robot
to each subgoal
in sequence

Cartesian Trajectory Motion



Move the robot
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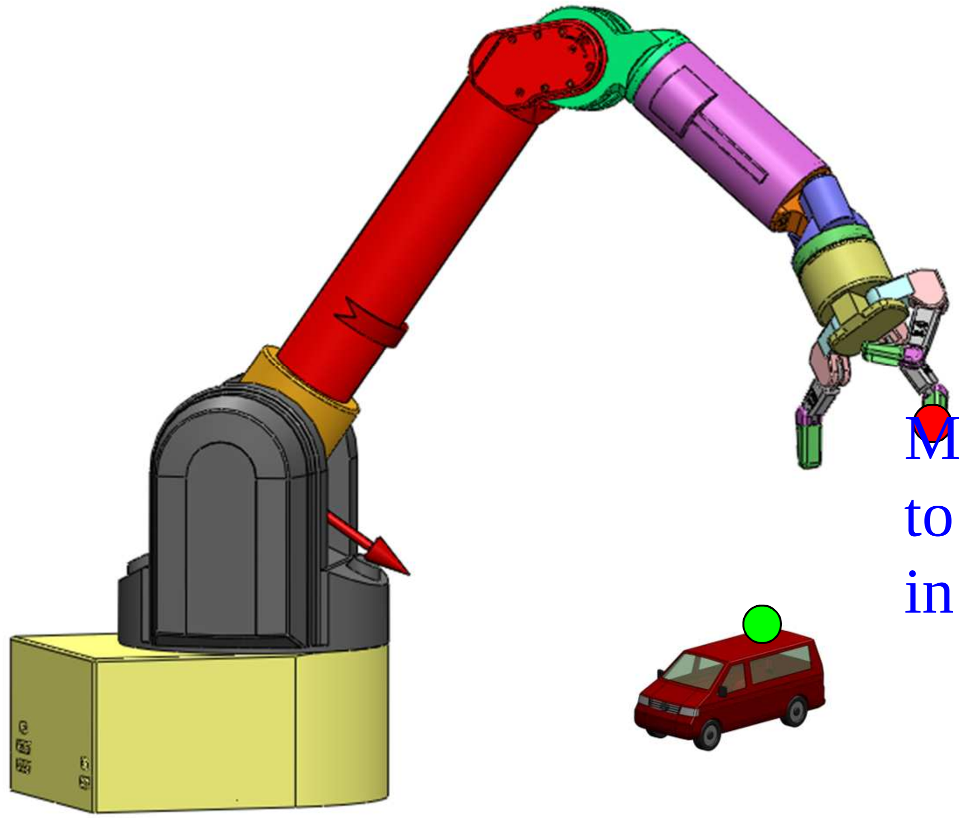
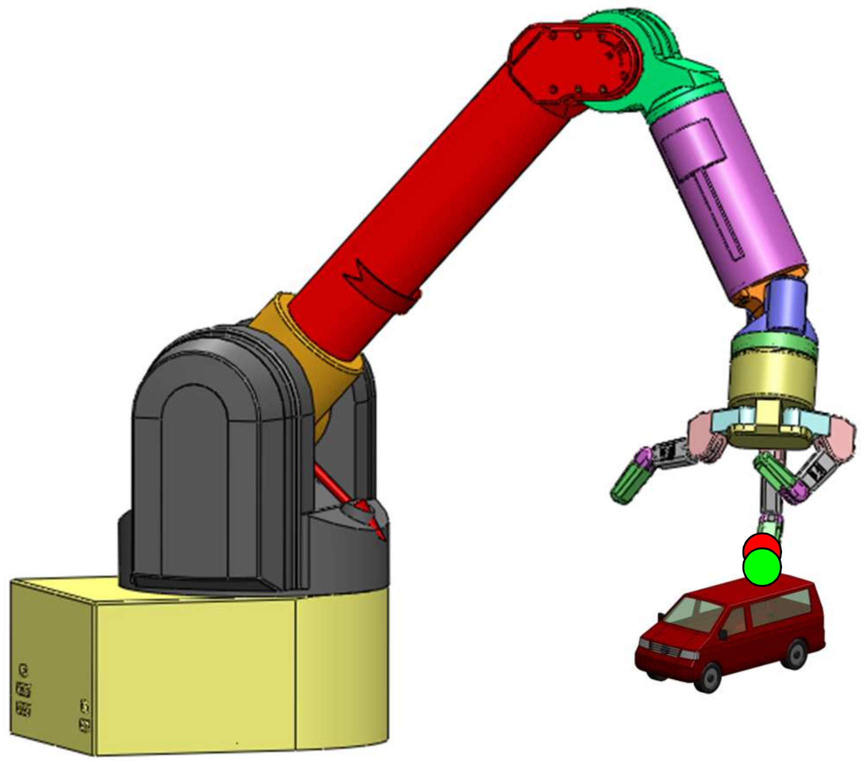
Cartesian Trajectory Motion



Move the robot
to each subgoal
in sequence

Cartesian Trajectory Motion



Iterate until convergence at final goal

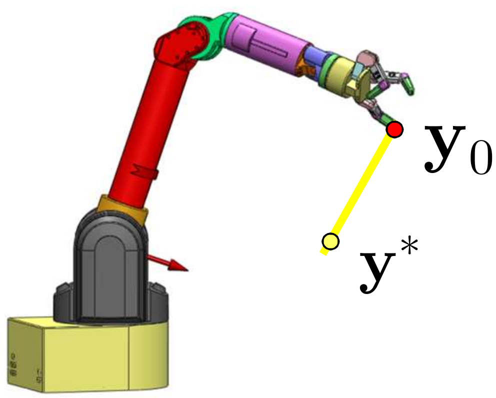
Numerical Inverse Kinematics

How do we find the Jacobian J(x)?

- Cartesian Location $\mathbf{y} = [x, y, z]^T \;=\; f(\mathrm{x})$
- Motor joint angles: $\mathbf{x} = [\, x_1, \quad x_2, \ldots \quad x_n\,]^T$
- Local linear model: $\Delta \mathbf{y} = \mathbf{J}(\mathbf{x})\Delta \mathbf{x}$
- Visual servoing steps:

1 Solve: $\mathbf{y}^* - \mathbf{y}_k = \mathbf{J}\Delta \mathbf{x}$

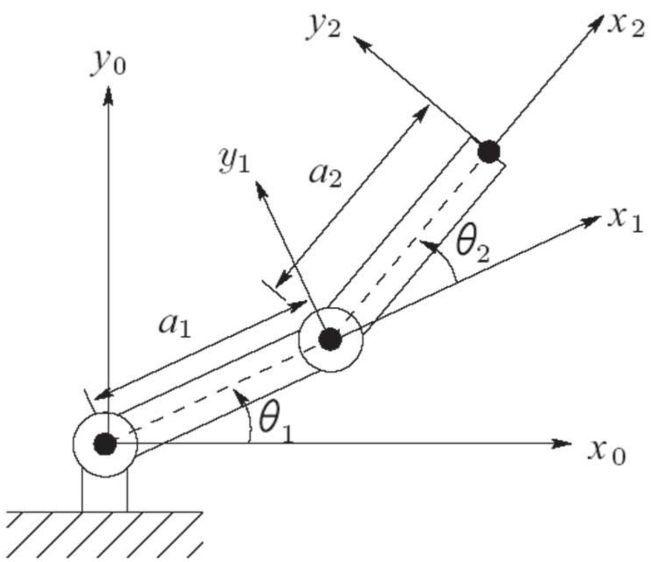
2 Update: $\mathbf{x}_{k+1} = \mathbf{x}_k + \Delta \mathbf{x}$



Jacobian

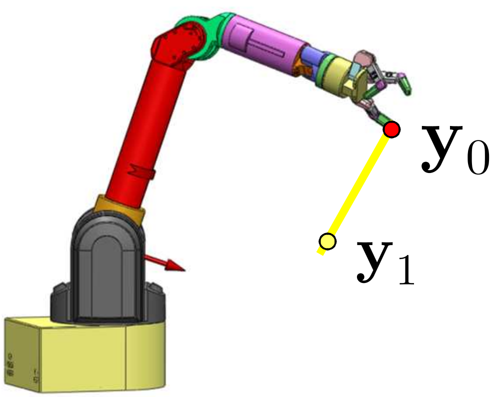
- We can analytically derive the Jacobian from the forward kinematics
- EX: two link manipulator
  - Analytic Jacobian J is:

$$J(q)=\begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$



Jacobian

- We can also numerically compute the Jacobian in various ways based on the Secant constraint
  1. Move to joint angles $\mathbf{x} = [\, x_1, \quad x_2, \ldots \quad x_n \,]^T$
  2. Fwd kin gives Eucl pos $\mathbf{y} = [x, y, z]^T \quad = \quad f(\mathrm{x})$
  3. Relative movements $\Delta\mathbf{y}, \Delta\mathbf{x}$
  4. Secant constraint for J: $\Delta\mathbf{y} = \mathbf{J}(\mathbf{x})\Delta\mathbf{x}$



Find J Method 1:
Test movements along basis

- Remember: J is unknown m by n matrix
  - For position only: 3x3, position and orientation 6x6 or mx6

$$\mathbf{J} = \begin{pmatrix} \frac{\partial \mathrm{f}_1}{\partial x_1} & \cdots & \frac{\partial \mathrm{f}_1}{\partial x_n} \\ \vdots & \ddots & \\ \frac{\partial \mathrm{f}_m}{\partial x_1} & & \frac{\partial \mathrm{f}_\mathrm{m}}{\partial x_n} \end{pmatrix}$$

$$\Delta\mathbf{x}_1 = [1, 0, \ldots, 0]^T$$
$$\Delta\mathbf{x}_2 = [0, 1, \ldots, 0]^T$$
$$\vdots$$
$$\Delta\mathbf{x}_n = [0, 0, \ldots, 1]^T$$

- Do test movements
- Finite difference:

$$\mathbf{Jt} \quad \left(\begin{bmatrix} \vdots \\ \Delta\mathbf{y}_1 \\ \vdots \end{bmatrix} \begin{bmatrix} \vdots \\ \Delta\mathbf{y}_2 \\ \vdots \end{bmatrix} \cdots \begin{bmatrix} \vdots \\ \Delta\mathbf{y}_n \\ \vdots \end{bmatrix} \right)$$

Find J Method 2: Secant Constraints

- Constraint along a line:
- Defines m equations

$$\Delta\mathbf{y} = \mathbf{J}\Delta\mathbf{x}$$

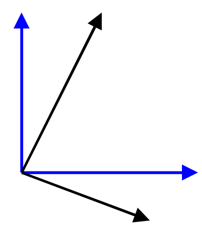
- Collect n arbitrary, but different measures y
- Solve for J

$$\begin{pmatrix} \begin{bmatrix} \cdots & \Delta\mathbf{y}_1^T & \cdots \end{bmatrix} \\ \begin{bmatrix} \cdots & \Delta\mathbf{y}_2^T & \cdots \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \cdots & \Delta\mathbf{y}_n^T & \cdots \end{bmatrix} \end{pmatrix} = \begin{pmatrix} \begin{bmatrix} \cdots & \Delta\mathbf{x}_1^T & \cdots \end{bmatrix} \\ \begin{bmatrix} \cdots & \Delta\mathbf{x}_2^T & \cdots \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \cdots & \Delta\mathbf{x}_n^T & \cdots \end{bmatrix} \end{pmatrix} \mathbf{J}^T$$

Find J Method 3: Recursive Secant Constraints Broydens method

- Based on initial J and one measure pair $\Delta \mathbf{y}, \Delta \mathbf{x}$
- Adjust J s.t. $\Delta \mathbf{y} = \mathbf{J}_{k+1} \Delta \mathbf{x}$
- Rank 1 update:

$$\hat{J}_{k+1} = \hat{J}_k + \frac{(\Delta \mathbf{y} - \hat{J}_k \Delta \mathbf{x}) \Delta \mathbf{x}^T}{\Delta \mathbf{x}^T \Delta \mathbf{x}}$$

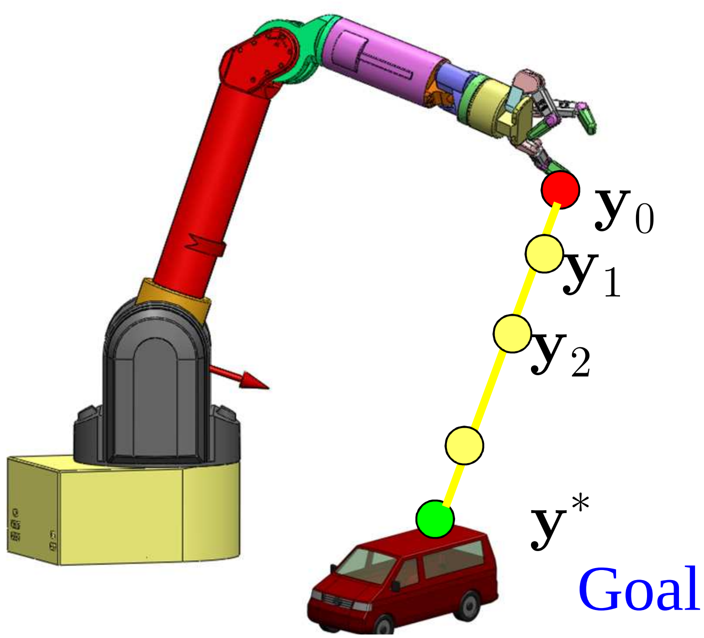


- Consider rotated coordinates:
  – Update same as finite difference for n orthogonal moves $\Delta \mathbf{x}$

Numerical Inverse Kinematics

repeat

1. Solve for motion: $[\mathbf{y}^* - \mathbf{y}_k] = \mathbf{J_k}\Delta\mathbf{x}$
2. Move robot joints: $\mathbf{x}_{k+1} = \mathbf{x}_k + \Delta\mathbf{x}$
3. Read actual Cartesian move $\Delta\mathbf{y}$
4. Update Jacobian: $\hat{J}_{k+1} = \hat{J}_k + \frac{(\Delta\mathbf{y} - \hat{J}_k\Delta\mathbf{x})\Delta\mathbf{x}^T}{\Delta\mathbf{x}^T\Delta\mathbf{x}}$



Move the robot to each subgoal in sequence $\mathbf{y}_k$

Iterate until convergence at final goal

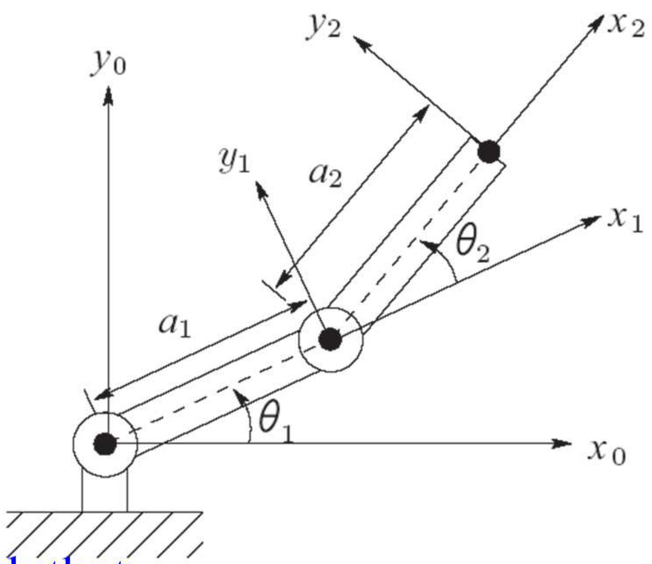
Singularities

- J singular ⇔ cannot solve eqs $[\mathbf{y}^* - \mathbf{y}_k] = \mathbf{J}\Delta\mathbf{x}$
- Definition: we say that any configuration in which the rank of *J* is less than its maximum is a singular configuration
  - i.e. any configuration that causes *J* to lose rank is a singular configuration
- Characteristics of singularities:
  - At a singularity, motion in some directions will not be possible
  - At and near singularities, bounded end effector velocities would require unbounded joint velocities
  - At and near singularities, bounded joint torques may produce unbounded end effector forces and torques
  - Singularities often occur along the workspace boundary (i.e. when the arm is fully extended)

Singularities

- How do we determine singularities?
  - Simple: construct the Jacobian and observe when it will lose rank
- EX: two link manipulator
  - Analytic Jacobian J is:

$$J(q)=\begin{bmatrix} -a_1s_1-a_2s_{12} & -a_2s_{12} \\ a_1c_1+a_2c_{12} & a_2c_{12} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$



  - This loses rank if we can find some α such that columns are linearly dependent $J_1 = \alpha J_2$ for $\alpha \in \mathbf{R}$

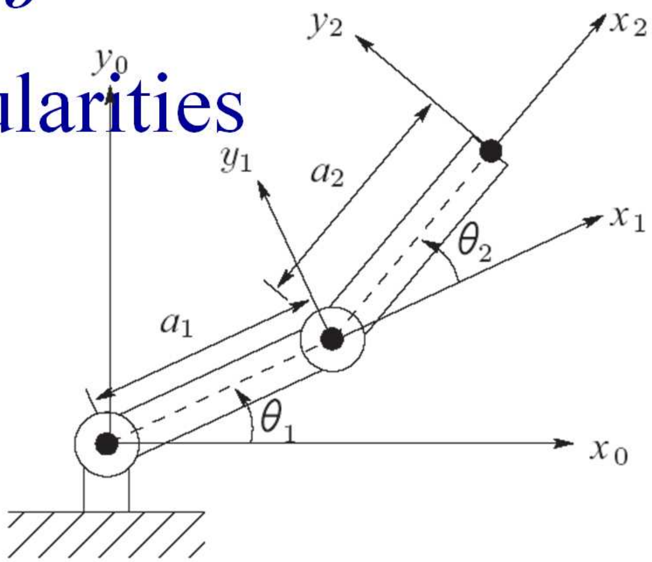
Singularities

- This is equivalent to the following:

$$a_1 s_1 + a_2 s_{12} = \alpha(a_2 s_{12})$$
$$a_1 c_1 + a_2 c_{12} = \alpha(a_2 c_{12})$$

- Thus if $s_{12} = s_1$, we can always find an α that will reduce the rank of J
- Thus $\theta_2 = 0, \pi$ are two singularities

$$\alpha = \frac{a_1 + a_2}{a_2} \qquad \alpha = \frac{a_2 - a_1}{a_2}$$

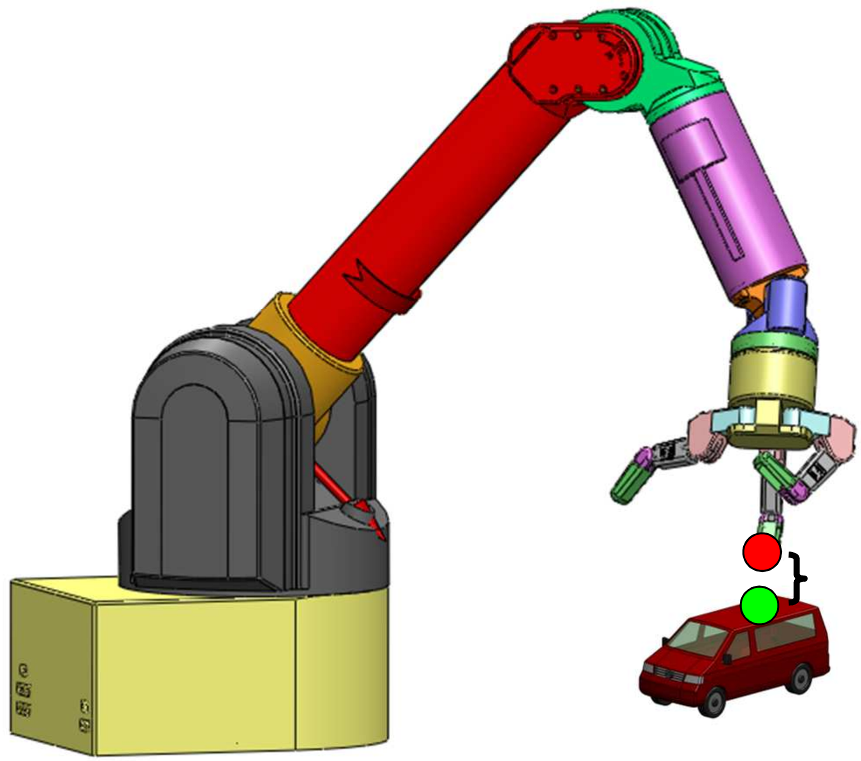


Determining Singular Configurations

- In general, all we need to do is observe how the rank of the Jacobian changes as the configuration changes
- Can study analytically
- Or numerically: Singular if eigenvalues of square matrix 0, or singular values of rectangular matrix zero. (Compute with SVD), or condition number tends to infinity.

How accurate is the movement?

- Does the robot always reach the goal?

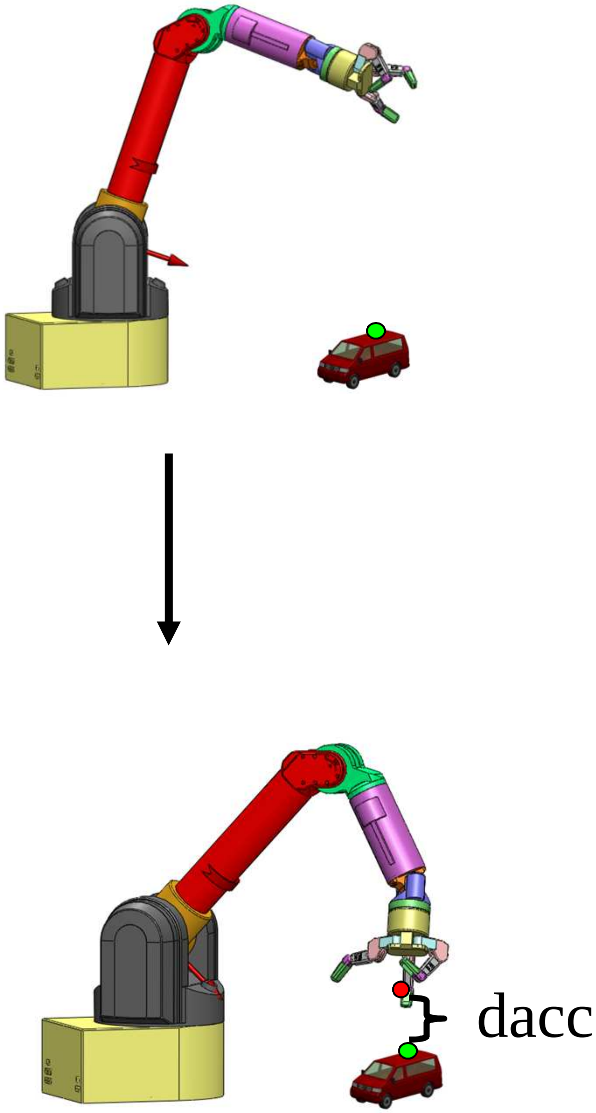


Final position error dy

Accuracy, Repeatability and Resolution

Accuracy:

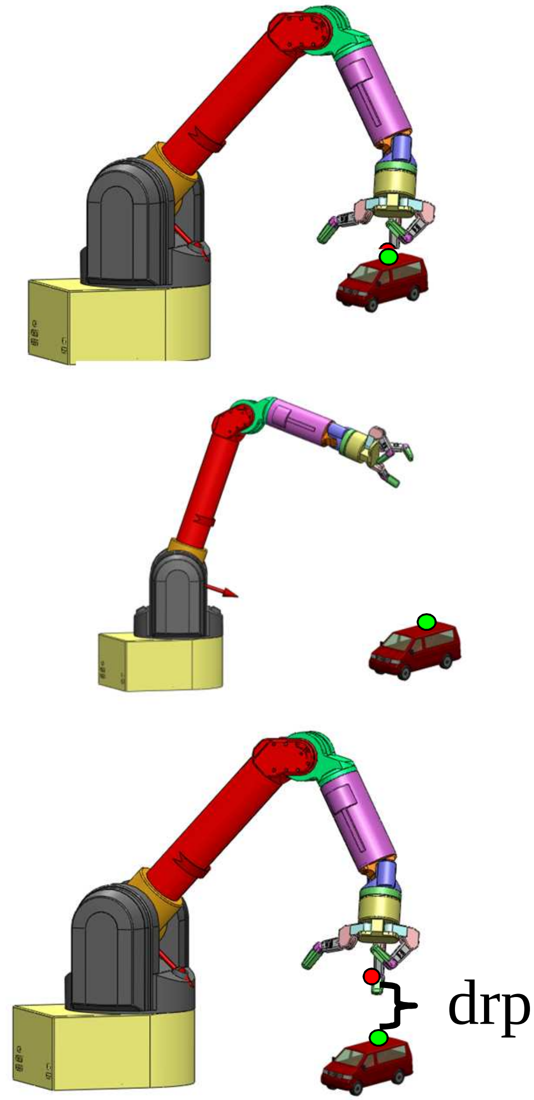
- Start far away
- Move a long distance
- Measure error dacc



Accuracy, Repeatability and Resolution

Repeatability:

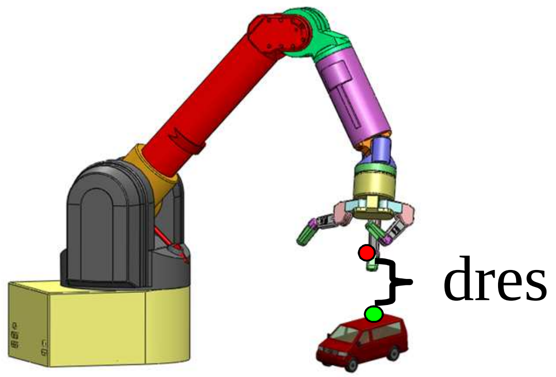
- Start at goal
- Move away a long distance
- Move back to goal
- Measure error drp

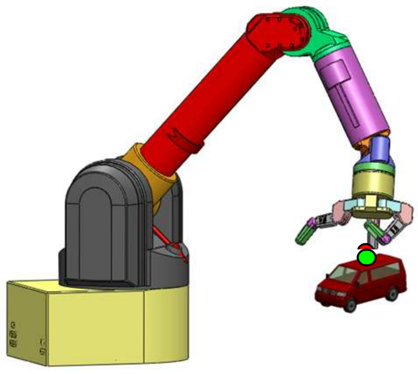


Accuracy, Repeatability and Resolution

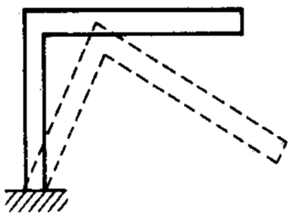
Resolution:

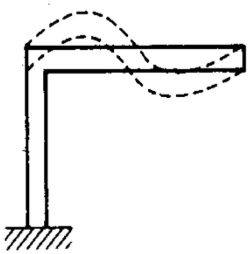
- The smallest incremental distance a robot can move.
- Typically limited by joint encoder resolution.





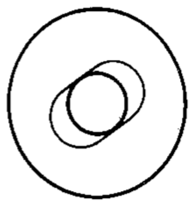
Points potentially weak in mechanical design

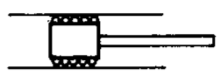


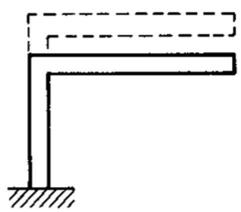


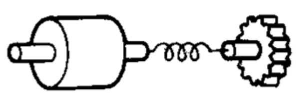
Weak points	Mechanical correction
Permanent deformation of the whole structure and the components	• Increase rigidness • Weight reduction • Counterweight
Dynamic deformation	• Increase rigidness • Reduction of the mass to move • Weight distribution
Backlash	• Reduce gear clearances • Use more rigid transmission elements

Points potentially weak in the mechanical design





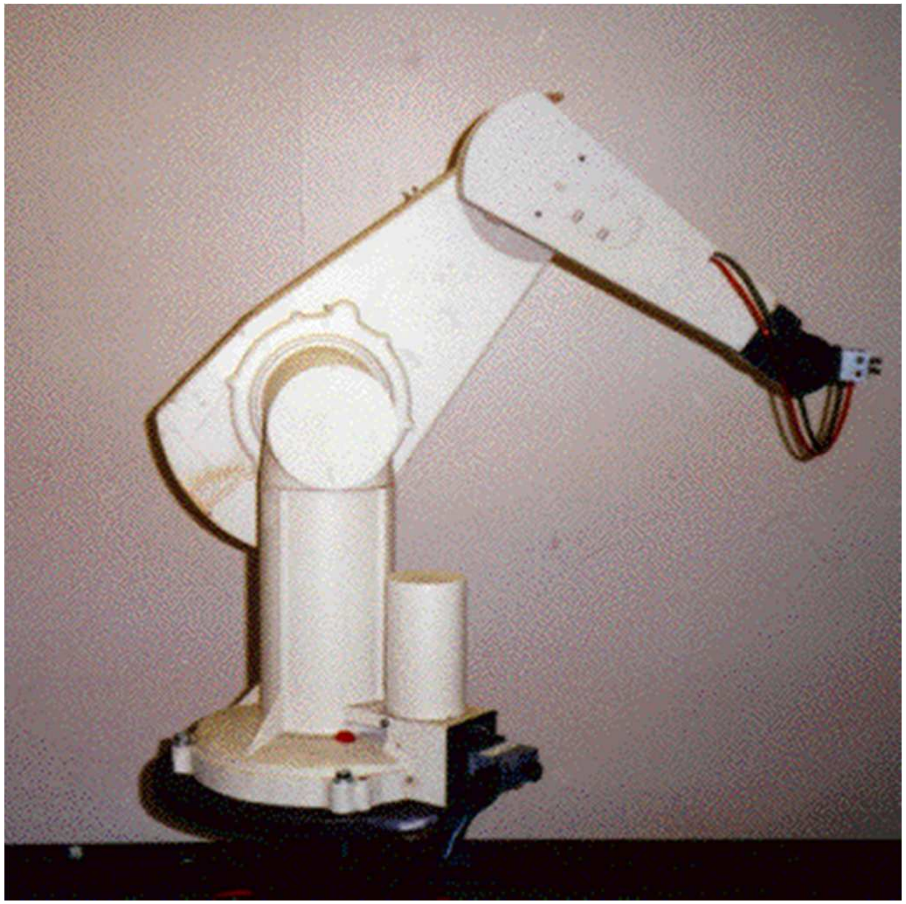




Weak points	Mechanical correction
Axes clearance	• Use pre stressed axes
Friction	• Improve clearance in axes • Increase lubrication
Thermal effects	• Isolate heat source
Bad transducers connection	• Improve mechanical connection • Search for a better location • Protect the environment

Robot Morphology:

An classic arm - The PUMA 560

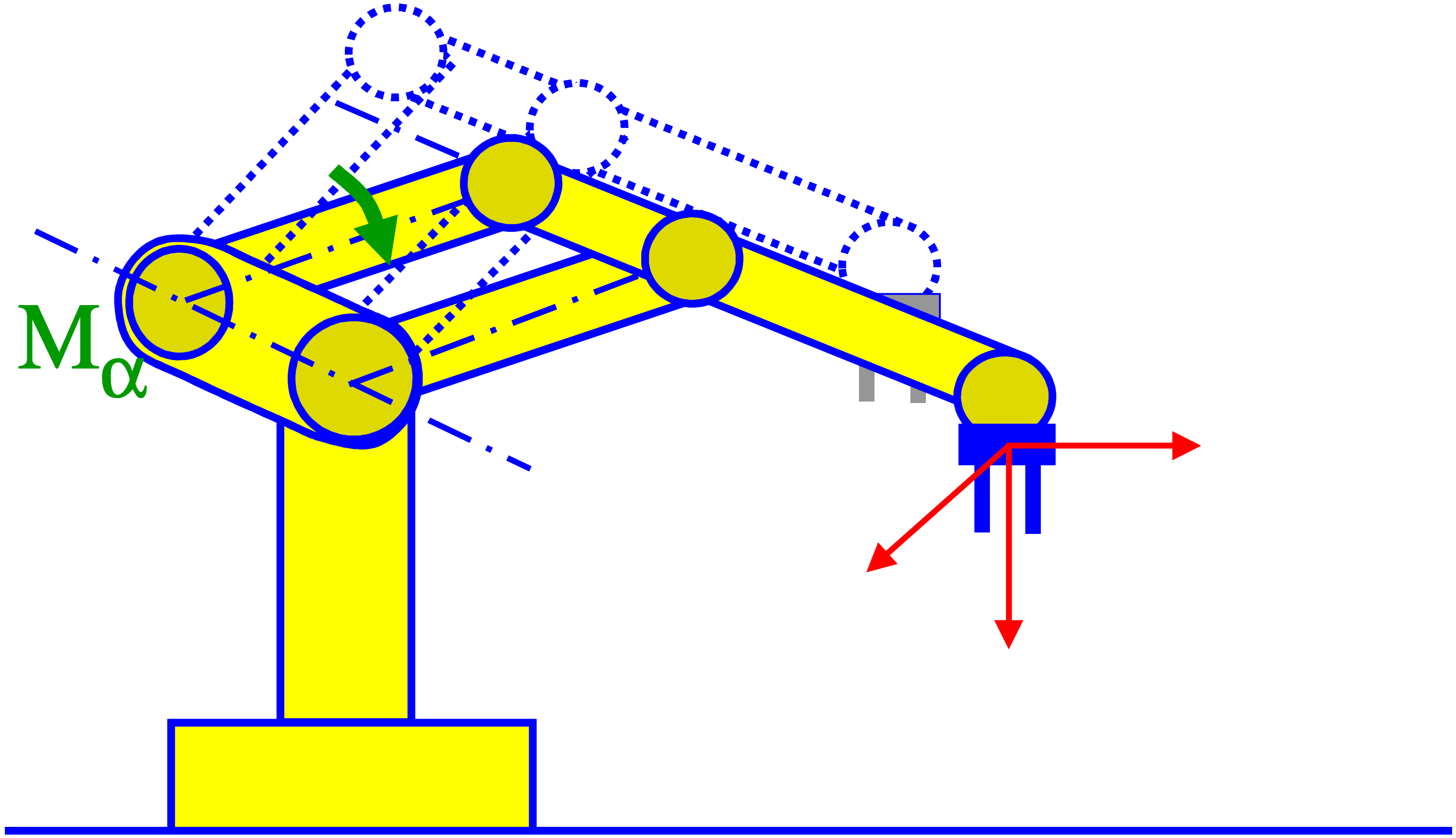


- Accuracy
  - Factory about 10mm
  - Calibrated 1-5mm
- Repeatability 1mm
- Resolution 0.1mm

An modern arm - The Barrett WAM

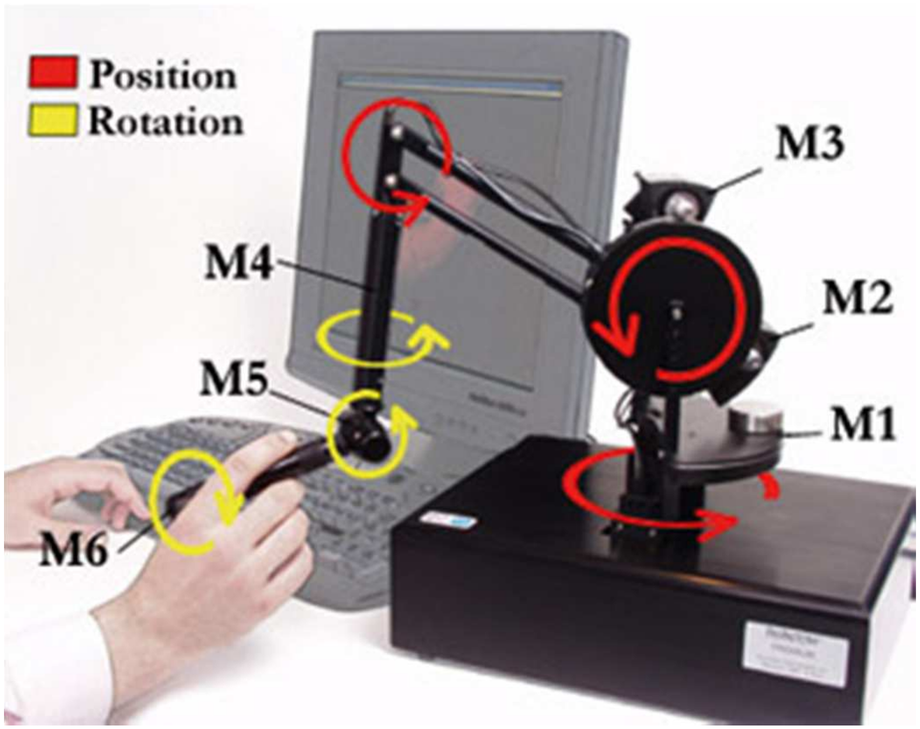
- Resolution and repeatability similar or better than PUMA
- Accuracy worse due to lighter, more flexible linkage
- But can move faster
- Needs external feedback (e.g. camera vision) for accurate motions

5 – Bar linkage

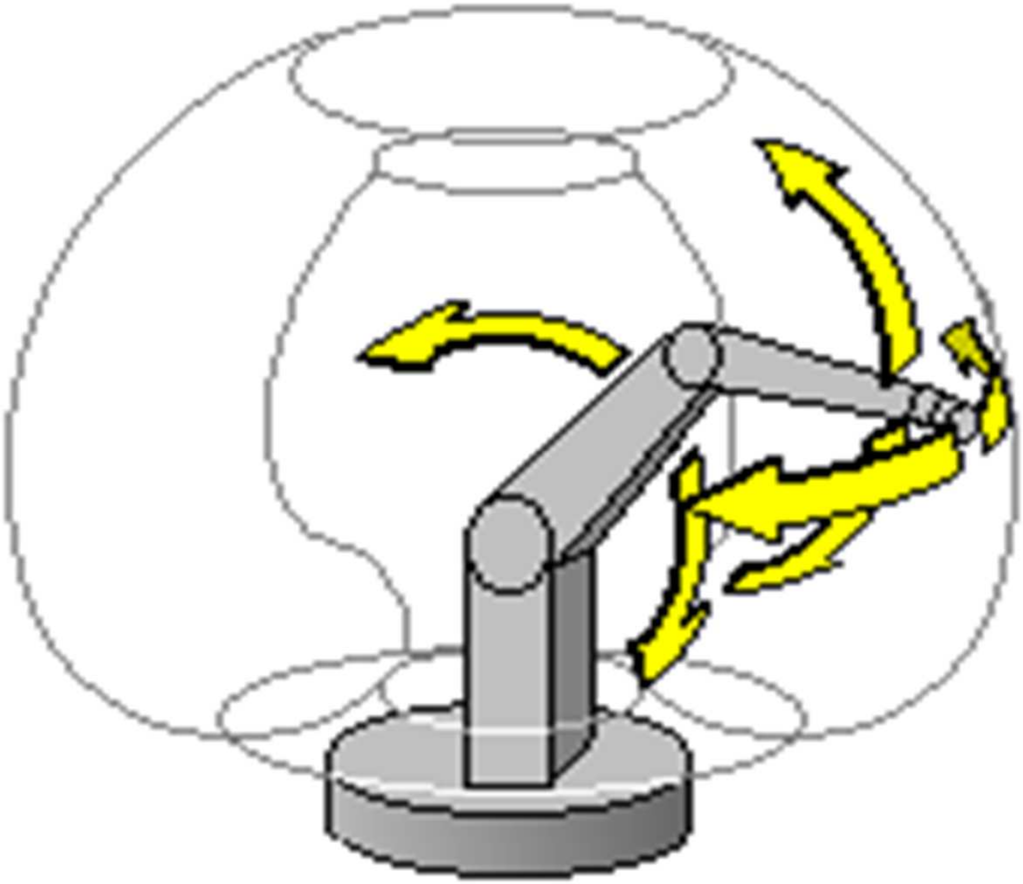


Can build this with LEGO!

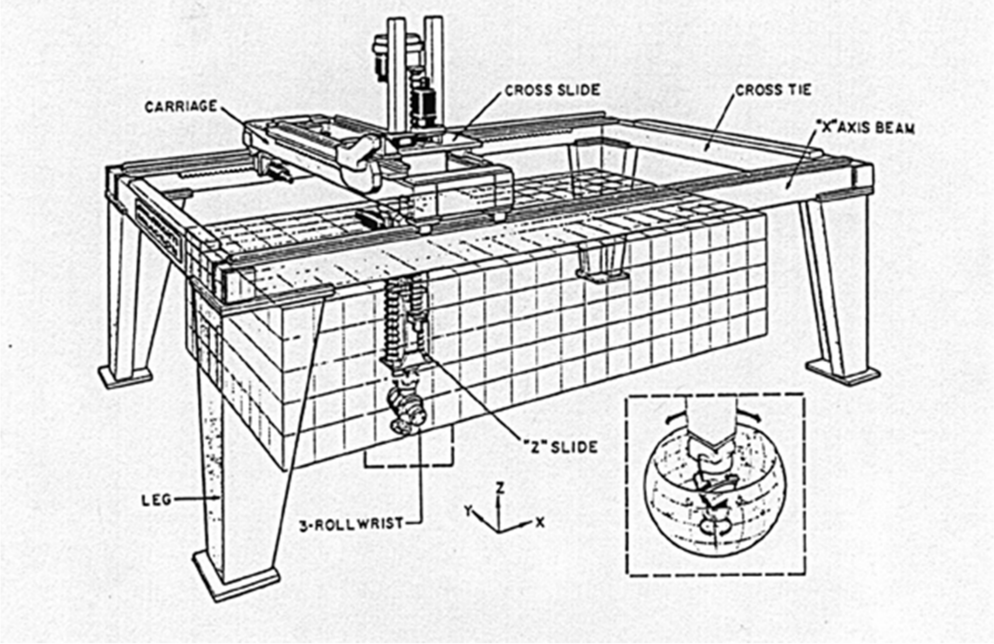
Parallel linkage
Sensable Phantom



Workspace of a typical serial arm



Cartesian robot



Typical performance

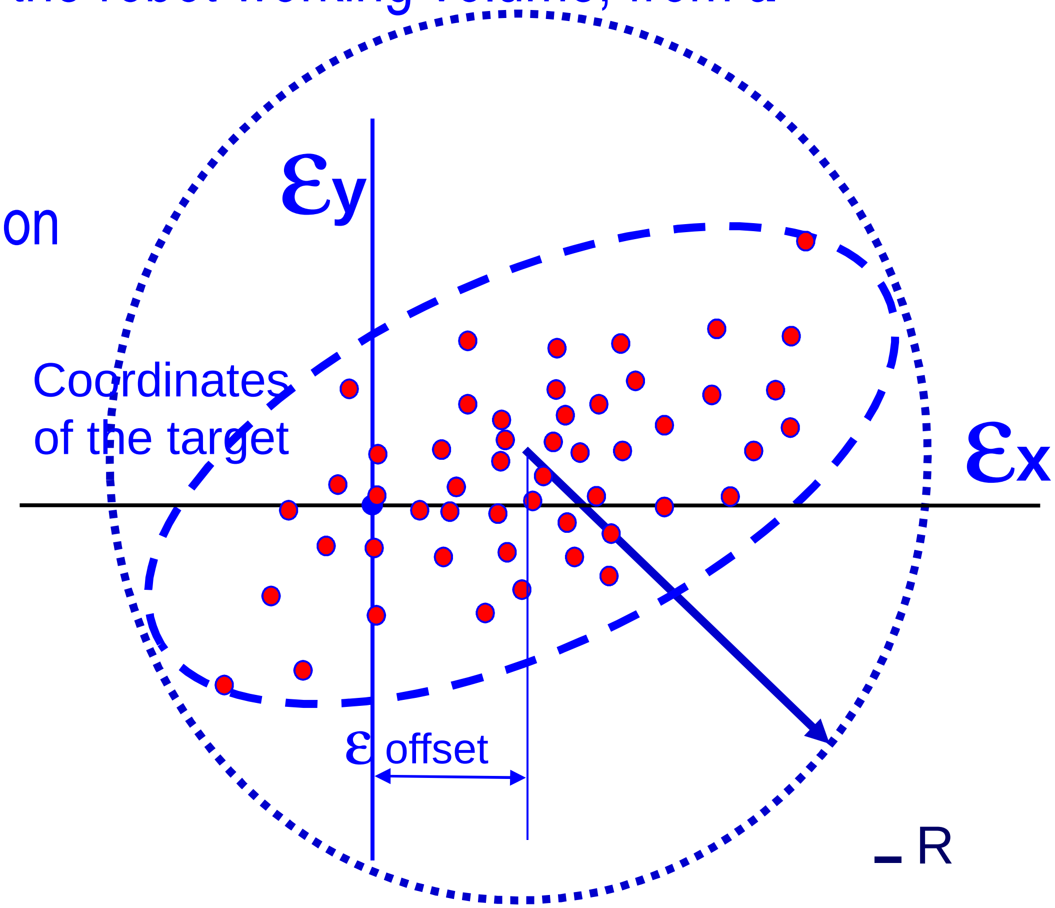
- Accuracy 0.1-1mm
- Repeatability: 0.01mm
- Resolution: 0.01mm
- Drawbacks
  - Large, heavy
  - Workspace “inside” the robot

Measuring Accuracy

Accuracy

- Capacity to place the end effector into a given position and orientation (pose) within the robot working volume, from **a random** initial position.

ε increases with the motion distance



Measure:

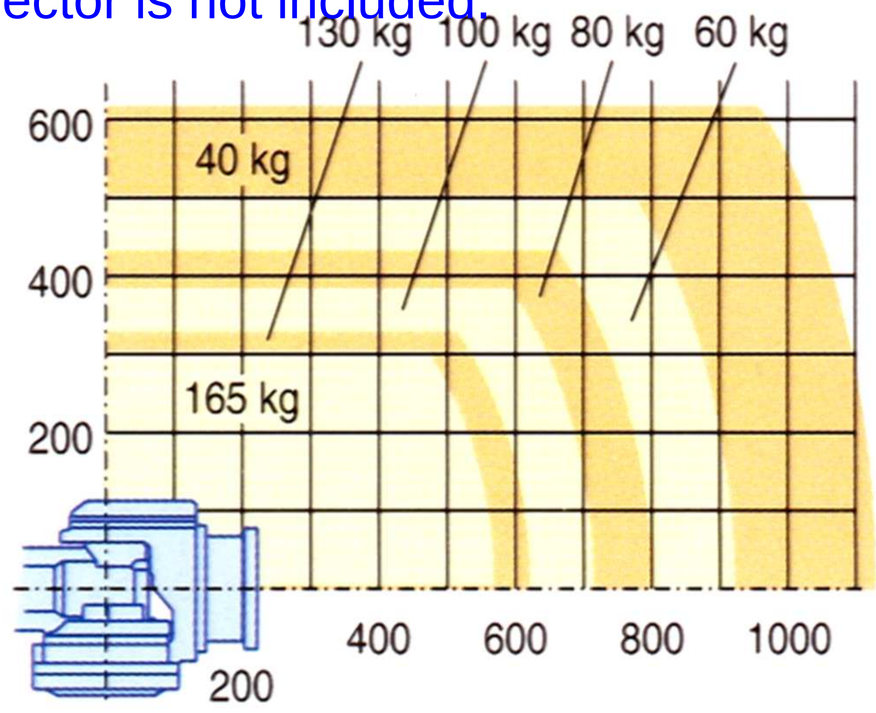
- Sample many start and end positions
- Characterize errors:
  - systematic ε (offset)
  - random

Dynamic Characteristics

Payload:

- The load (in Kg) the robot is able to transport in a continuous and precise way (stable) to the most distance point
- The values usually used are the maximum load and nominal at acceleration = 0
- The load of the End-Effector is not included.

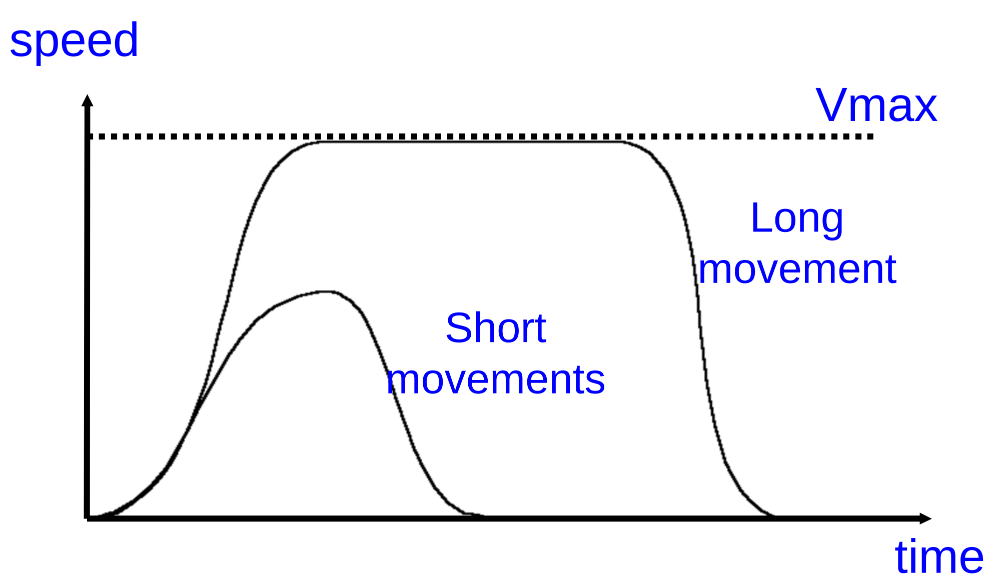
Example of Map of admitted loads, in function of the distance to the main axis



Dynamic Characteristics

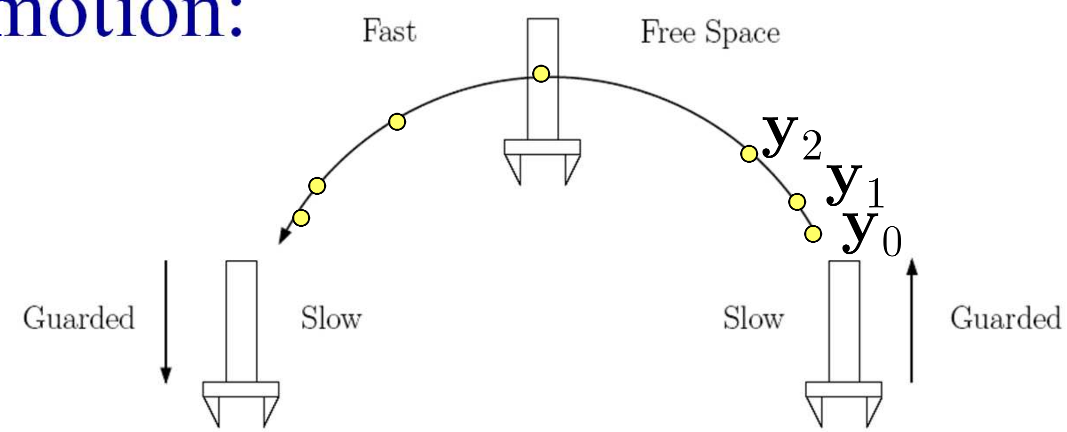
Velocity

- Maximum speed (mm/sec.) to which the robot can move the End-Effector.
- It has to be considered that more than a joint is involved.
- If a joint is slow, all the movements in which it takes part will be slowed down.
- For shorts movements acceleration matters more.



Path/trajectory planning

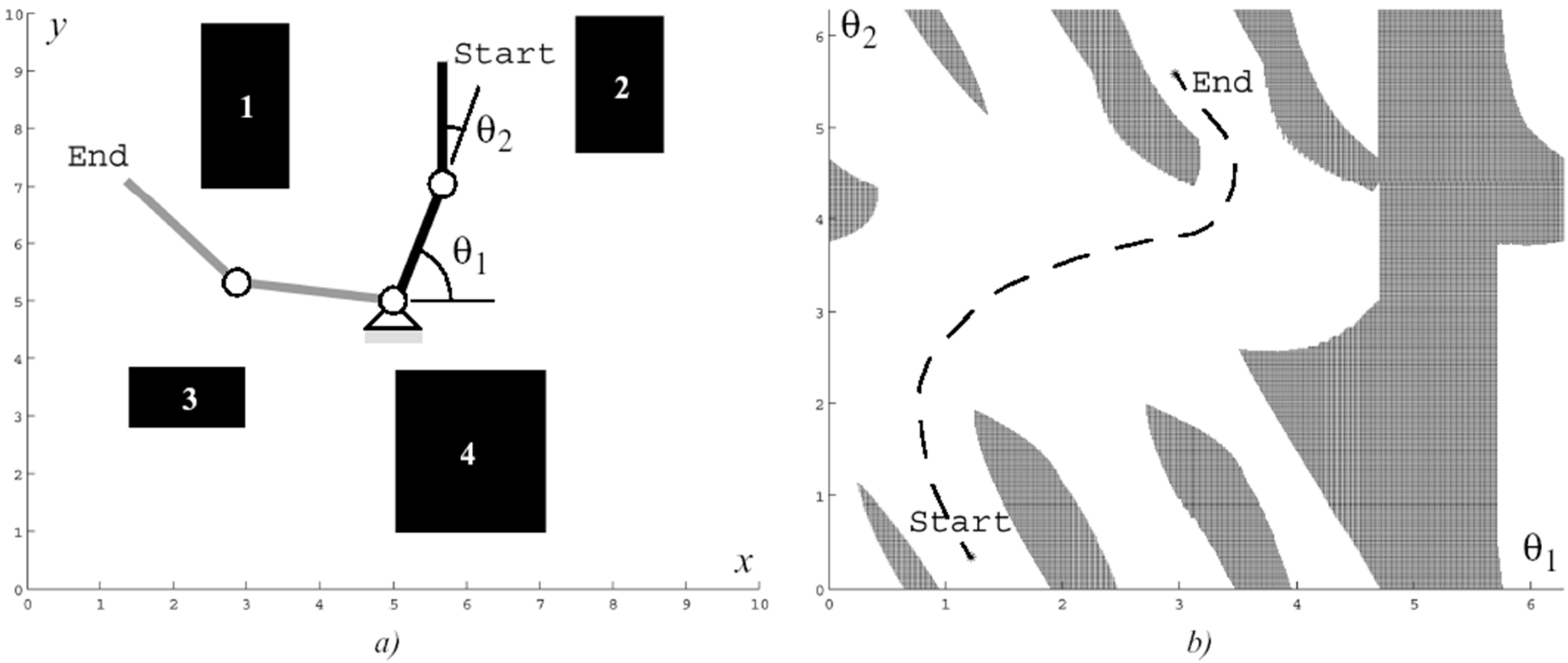
- Split the workspace into areas of fast and slow guarded motion:



- In practice robot has to smoothly accelerate.
- Can create position/velocity profiles with linear or higher order polynomials/splines.

Path/trajectory planning

- With many obstacles may need motion planning algorithms (Potential fields, RRT etc -- later)



Conclusions:

- Different architectures have different kinematics and different accuracy.
  - Serial arms: Slender, agile but somewhat inaccurate
  - Cartesian (x,y,z- table): Very accurate, but bulky
- Accuracy can be improved by:
  - Cartesian calibration
  - Visual or other sensory feedback (next in course)