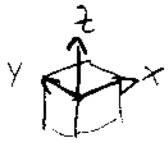
Notes : LAB 2 : Robot arm Kinematics and Path planning

(1) Location → Mobile Robots } odometry → SLAM (Simultaneous localization and mapping.
→ Robot Arms.

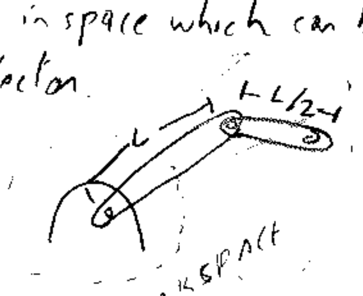
WIKIPEDIA:
→ build a map within an unknown environment (without a priori knowledge)
or update map within a known environment
while at the same time keeping track of their current location.
→ Kalman filter
Particle filter
Monte Carlo localization
⋮

* Robotics concerned with location of objects in 3D space.
* Objects has position and orientation.



"DARPA GRAND CHALLENGE" } 150 miles
1st 2004 → Mojave Desert
2d 2005 } Stanley → Smithsonian National Museum of American History (Washington)
"Third" 2007 → Urban Challenge
CMU 2007 Chevy Tahoe (Boss)
Stanford (grüner) Volkswagen
2012 → "Robotics challenge"
└ 2014 } 1. Drive utility vehicle at the site
2. Open a door enter a building
5. Climb industrial ladder.
7. Locate and close a valve near leaking pipe.
8. Replace a
6. Use a tool to break through a concrete panel.

(2) Kinematics:
- Robot Manipulator: links inter-connected by joints
- Subspace: is a subset of the subspace End-effector.
- Workspace: Points in space which can be reached by the end-effector.
→ "plane"



(3)
- Forward & inverse Kinematics

world coordinates = F(robot variables) } Easy to compute.

$$x = x(\theta_1, \theta_2)$$
$$y = y(\theta_1, \theta_2)$$

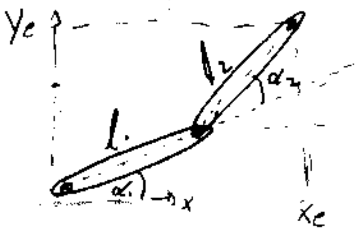
- Jacobian: time derivative of robot variables.

Inverse Kinematics:
robot variables = F(world coordinates) } Inverse problem has difficulties. Inversion may not be unique.

$$\theta_1 = \theta_1(x, y)$$
$$\theta_2 = \theta_2(x, y)$$

(4) DYNAMICS "FORCE REQUIRE TO CAUSE MOTION"
Forward dynamics → Given forces, work out acceleration
Inverse dynamics → Given acceleration work out forces

Forward Kinematics



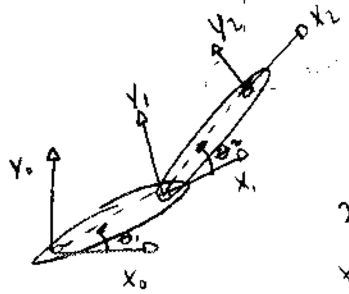
$X_e = l_1 \cos(\alpha_1) + l_2 \cos(\alpha_1 + \alpha_2)$

$Y_e = l_1 \sin(\alpha_1) + l_2 \sin(\alpha_1 + \alpha_2)$

$f_1(\theta_1, \theta_2) = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) - X_e$

$f_2(\theta_1, \theta_2) = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) - Y_e$

non-linear equations



$x_2 \cdot x_0 = \cos(\theta_1 + \theta_2)$; $y_2 \cdot x_0 = -\sin(\theta_1 + \theta_2)$

$x_2 \cdot y_0 = \sin(\theta_1 + \theta_2)$; $y_2 \cdot y_0 = \cos(\theta_1 + \theta_2)$

Forward Kinematic 2 DOF.
for 6 DOF equations are complex
Denavit-Hartenberg

Homogeneous coordinates & homogeneous transformations

Inverse Kinematics } forward Kinematics equations are nonlinear. } no unique solution in general.

* out of reach? } no solution
* Two solutions
* One solution, manipulator fully extended.
* infinite solutions (0,0) point $l_1 = l_2$ → "origin"

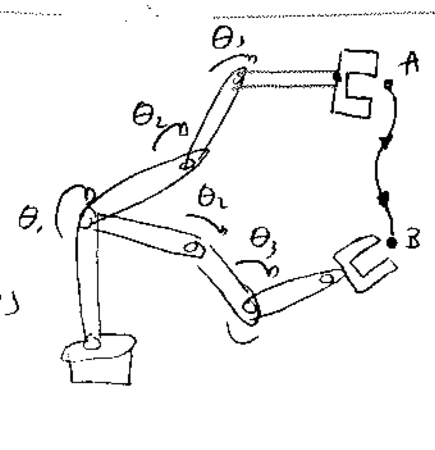
Close-form Solution
* Algebraic
* Geometric method

Numerical Solution

⑤ Trajectory Generation

Get from A to B

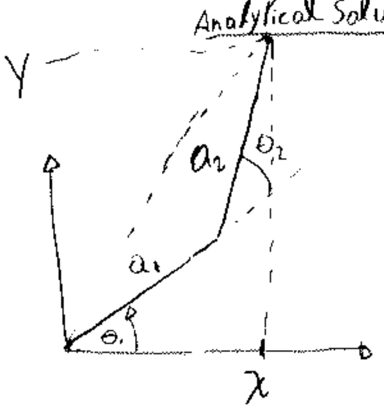
* In Robot coordinates $\theta_A \to \theta_B$
* Planning in robot coordinates is easy but we lose visualization
* Additional constraint.
  smoothness
  dynamic limitations
  obstacles



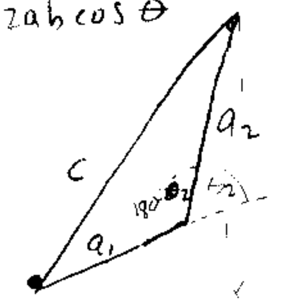
PAG 3

Inverse Kinematics

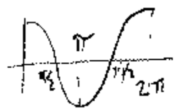
Analytical Solution



$c^2 = a^2 + b^2 - 2ab\cos\theta$



$\cos(\pi - x) = -\cos(x)$



$$x^2 + y^2 - a_1^2 - a_2^2 = -2a_1a_2\cos(180 - \theta_2)$$

$$x^2 + y^2 - a_1^2 - a_2^2 = 2a_1a_2\cos(\theta_2)$$

$$\cos(\theta_2) = \frac{x^2 + y^2 - a_1^2 - a_2^2}{2a_1a_2} = D$$

$$\theta_2 = \cos^{-1}(D)$$

"Better way" $\sin^2\theta_2 + \cos^2\theta_2 = 1$

$$\sin(\theta_2) = \pm\sqrt{1 - D^2}$$

$$\theta_2 = \tan^{-1}\left(\frac{\pm\sqrt{1 - D^2}}{D}\right) \quad \frac{\sin(\theta_2)}{\cos(\theta_2)}$$

ADVANTAGE $\pm$ } both solutions are recover

elbow up

elbow down

$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\underbrace{\left(\frac{a_2\sin\theta_2}{a_1 + a_2\cos\theta_2}\right)}$$

notice
depends θ_2
(make sense physically
since we would expect
different value for θ_1
depending on which
solution of θ_2
is chosen for

Velocity Kinematics

$$\dot{x} = -a_1\sin\theta_1 \cdot \dot{\theta}_1 - a_2\sin(\theta_1 + \theta_2)(\dot{\theta}_1 + \dot{\theta}_2)$$

$$\dot{y} = a_1\cos\theta_1 \cdot \dot{\theta}_1 + a_2\cos(\theta_1 + \theta_2)(\dot{\theta}_1 + \dot{\theta}_2)$$

$$\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} -a_1\sin\theta_1 - a_2\sin(\theta_1 + \theta_2) & -a_2\sin(\theta_1 + \theta_2) \\ a_1\cos\theta_1 + a_2\cos(\theta_1 + \theta_2) & a_2\cos(\theta_1 + \theta_2) \end{bmatrix}\dot{\theta}$$

$$= J\dot{\theta}$$

$$\dot{\theta} = J^{-1}\dot{x}$$

joint velocities

$$J^{-1} = \frac{1}{a_1a_2\sin\theta_2}\begin{bmatrix} a_2\cos(\theta_1 + \theta_2) & a_2\sin(\theta_1 + \theta_2) \\ -a_1\cos\theta_1 - a_2\cos(\theta_1 + \theta_2) & -a_1\sin\theta_1 - a_2\sin(\theta_1 + \theta_2) \end{bmatrix}$$

singular conf.

$\theta_2 = 0$; $\theta_2 = \pi$.