Non-linear systems, continuation method

Consider a system depending on a parameter λ:

$$F(X;\lambda) = 0$$

Assume that the solution is "easy" to find for $\lambda = 0$ and the interesting "difficult" solution occurs for $\lambda = 1$.

Solve for a sequence

$$0 = \lambda_0 < \lambda_1 < \ldots < \lambda_m = 1$$

A small step size might be required for convergence.

Non-linear systems, continuation method

The parameter interval $\lambda \in [0, 1]$ defines a *solution path* in R^n . The continuation method works if the Jacobian is non-singular along this path.

Problems occur at singular points:

- Crossing of solution paths.
- Limit points, i.e., solution ceases to exist.

Arc length continuation can be used to pass singular points.

Non-linear systems, continuation method

Time savings in continuation methods:

- Extrapolate $X(\lambda_{k-1})$ and $X(\lambda_k)$ to find $X^{(0)}(\lambda_{k+1})$.
- Do not solve intermediate steps accurately.
- Use fixed Jacobian for several step.