

Example :

$$R(\theta_1) = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 1 & 0 \\ 0 & 0 & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S(\theta_2) = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 0 \\ \cos(\theta_2) & 0 & \sin(\theta_2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{If } f(\theta_1, \theta_2) = R(\theta_1) S(\theta_2) [0, 0, 0, 1]^T$$

$$\text{Jacobian} = \begin{bmatrix} \frac{\partial f}{\partial \theta_1} & \frac{\partial f}{\partial \theta_2} \end{bmatrix} = \begin{bmatrix} \frac{\partial R(\theta_1)}{\partial \theta_1} \cdot S(\theta_2) \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} & R(\theta_1) \cdot \frac{\partial S(\theta_2)}{\partial \theta_2} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \end{bmatrix}$$

where

$$\frac{\partial R(\theta_1)}{\partial \theta_1} = \begin{bmatrix} -\sin(\theta_1) & -\cos(\theta_1) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial S(\theta_2)}{\partial \theta_2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \sin(\theta_2) & 0 & -\cos(\theta_2) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$