Chapter 7

Factorization methods, Recognition

7.1 Assume that 4 images of a rigid 7 point configuration $\mathbf{X}$ are taken with a camera that can be modelled as an affine camera, i.e. each camera is of the type

$$P \sim \begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} .$$

Show that the depth λ in the projection equation $\lambda \mathbf{x}_{ij} = P_i \mathbf{X}_j$ must be one. Convince yourself that it is possible to collect the projection equations for all points in all images as

$$\underbrace{\begin{bmatrix} \mathbf{x}_{1,1} & \dots & \mathbf{x}_{1,7} \\ \mathbf{x}_{2,1} & \dots & \mathbf{x}_{2,7} \\ \mathbf{x}_{3,1} & \dots & \mathbf{x}_{3,7} \\ \mathbf{x}_{4,1} & \dots & \mathbf{x}_{4,7} \end{bmatrix}}_{\hat{\mathbf{x}}} = \underbrace{\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}}_{\mathbf{P}} \underbrace{\begin{bmatrix} \mathbf{X}_1 & \dots & \mathbf{X}_7 \end{bmatrix}}_{\mathbf{X}} .$$

How could $\mathbf{X}$ and $\mathbf{P}$ be calculated from $\hat{\mathbf{x}}$?

7.2 Let three collinear points have coordinates

$$\mathbf{x} = \begin{bmatrix} a & b & c \\ 1 & 1 & 1 \end{bmatrix} .$$

Show that the ratio $(b-a)/(c-b)$ can be written as

$$\frac{\det \begin{bmatrix} a & b \\ 1 & 1 \end{bmatrix}}{\det \begin{bmatrix} b & c \\ 1 & 1 \end{bmatrix}}$$

Show that the ratio (as written above) is invariant under affine transformations

$$\mathbf{y} = T\mathbf{x} = \begin{bmatrix} d & e \\ 0 & 1 \end{bmatrix} \mathbf{x}$$

Hint: Use the determinant product rule on

$$\det \left(T \begin{bmatrix} a & b \\ 1 & 1 \end{bmatrix} \right)$$

.

7.3 Let four collinear points have coordinates

$$\mathbf{x} = \begin{bmatrix} a & b & x & y \\ 1 & 1 & 1 & 1 \end{bmatrix} .$$

Show that the cross ratio can be written as a

$$\frac{\begin{bmatrix} a & x \\ 1 & 1 \end{bmatrix} \begin{bmatrix} b & y \\ 1 & 1 \end{bmatrix}}{\begin{bmatrix} a & y \\ 1 & 1 \end{bmatrix} \begin{bmatrix} b & x \\ 1 & 1 \end{bmatrix}}$$

Show that the cross ratio (as written above) does not change if we multiply one of the culums with a non-negative scalar. Show that the cross ratio is invariant under projective transformations.

Chapter 8

Calibration

8.1 The point

$$\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}$$

the plane

$$\Pi = \begin{pmatrix} 0 \\ 1 \\ 0 \\ -2 \end{pmatrix}$$

the quadric (in its primal form)

$$\mathbf{Q} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -6 \end{pmatrix}$$

and camera matrix

$$\mathbf{P} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \end{pmatrix}$$

are given. What is the image of the point, plane and quadric?
A change of object coordinates is given as a transformation matrix

$$T = \begin{pmatrix} 2 & 0 & 0 & 4 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & 1 \end{pmatrix} .$$

Calculate the coordinate representation of the point, line, quadric and camera matrix in the new coordinates. What is the image of the point, plane and quadric after changing the object coordinate system? Has the image changed as we change object coordinate system?
A change of image coordinates is given as a transformation matrix

$$K = \begin{pmatrix} 100 & 0 & 50 \\ 0 & 100 & 50 \\ 0 & 0 & 1 \end{pmatrix} .$$

What is the image of the point, plane and quadric after changing the image coordinate system? What is the representation of the camera matrix after changing both object and image coordinate system?

8.2 A camera matrix is given as

$$P = \begin{pmatrix} 2 & 1 & -2 & 5 \\ 2 & -2 & 1 & 7 \\ 1 & 2 & 2 & 11 \end{pmatrix} .$$

Does the camera matrix represent a calibrated camera? A change of object coordinate system is given by

$$T = \begin{pmatrix} 2 & 0 & 0 & 4 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & 1 \end{pmatrix} .$$

Calculate the camera matrix in the new coordinate system. Does the camera matrix represent a calibrated camera?

8.3 An image conic is represented by a symmetric 3×3 matrix defined up to scale. What is the dimension of this manifold? Can you give at least one map $\mathbf{R}^d \to \omega$.