Projects

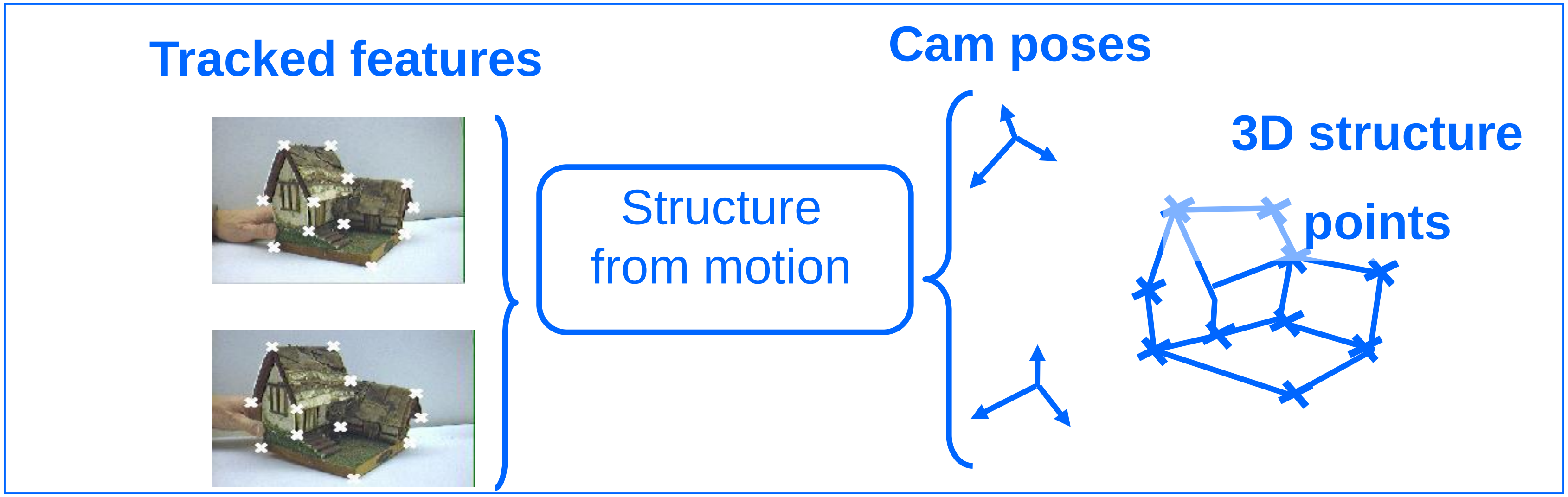
- Any Q/A or any help TA and I can provide?
- Feel free to engage in dialogue with TA/me on how to proceed
- Think about how to connect the project to course material.
- Interaction encouraged.
    - Attribute contributions to the people/sources

Light and Reflectance

Dana Cobzas Neil Birkbeck Martin Jagersand

most of course until now …

- SFM to reconstruct 3D points from 2D feature points (camera geometry, projective spaces …)
  - Feature correspondence : correlation, tracking assumes image constancy – constant illumination, no specularities, complex material
  - 10,100 or even 1000 3D points is not a complete scene or object model
- No notion of object surface
- No notion of surface properties (reflectance)

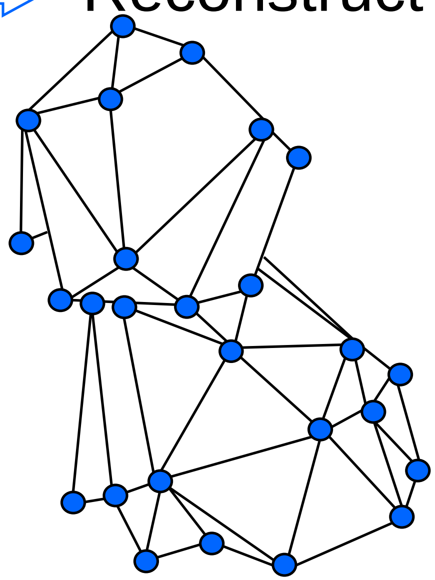


Now …

- View surface as a whole – different surface representations
- Consider interaction of surface with light – explicitly model light, reflectance, material properties

Reconstruct whole objects = surface (detailed geometry)

Reconstruct material properties = reflectance



SFM

Ex CapGui obj

Surface estimation

Reflectance estimation

Brief outline

- Image formation - camera, light,reflectance
- Radiometry and reflectance equation
- BRDF
- Light models and inverse light
- Shading, Interreflections

Lec 1

- Image cues – shading
    - Photometric stereo
    - Shape from shading
- Image cues – stereo
- Image cues – general reflectance

Lec 2

- Multi-view methods
    - Volumetric – space carving
    - Graph cuts
    - Variational stereo
    - Level sets
    - Mesh

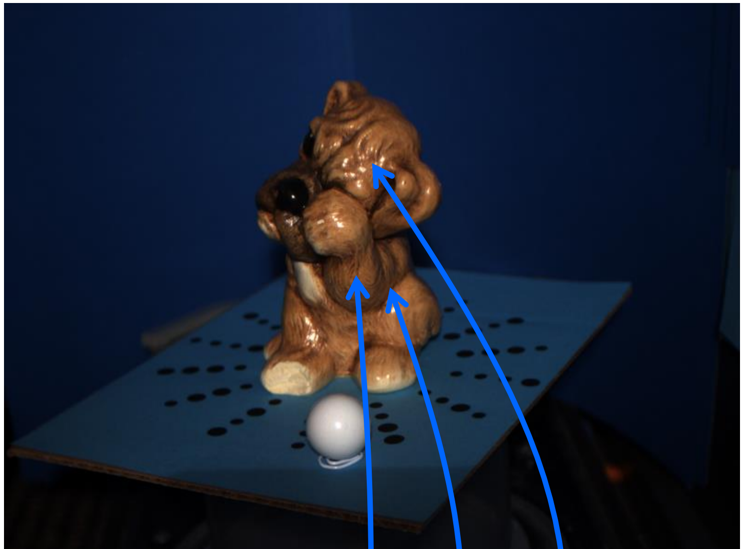
Lec 3,4

Lecture 1

Radiometry
Light and Reflectance

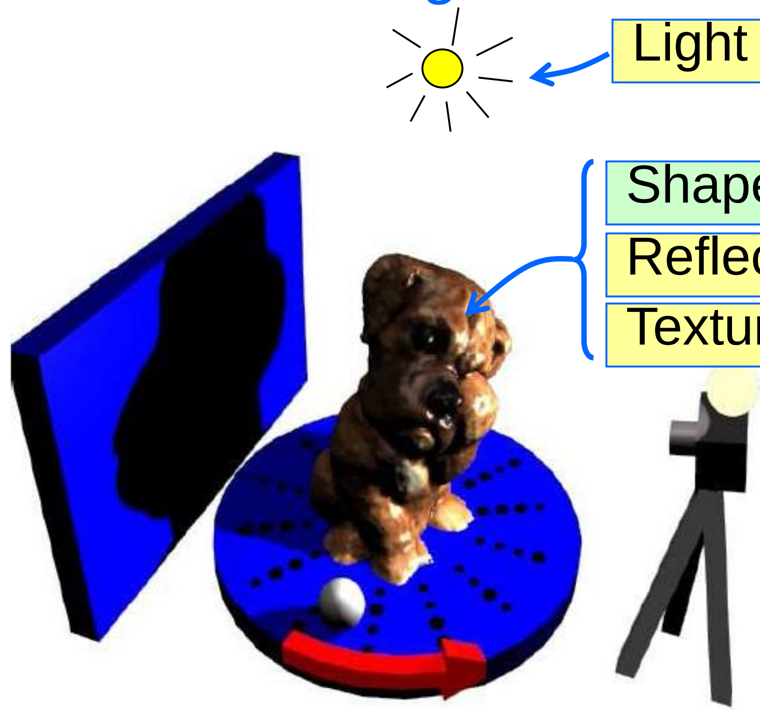
Image formation

Image



Shading
Shadows
Specular highlights
[Intereflections]
[Transparency]

Image formation



Light

Shape
Reflectance
Texture

Camera

Images 2D + [3D shape] ⇨ **Light** [+ Reflectance+Texture]

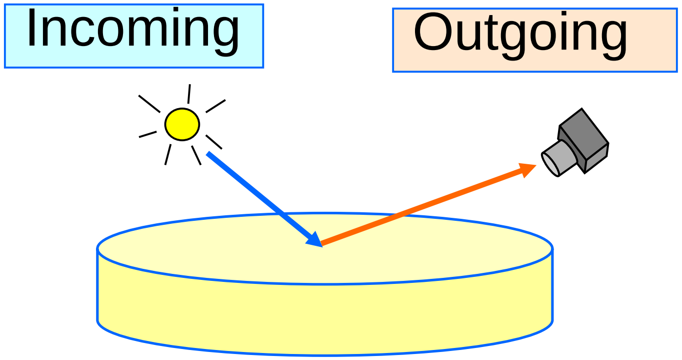
Summary

Various things we can model

1. Cameras
2. Radiometry and reflectance equation
3. BRDF – surface reflectance
    Lambertian BRDF
4. Light representation –
5. Image cues: shading, shadows, interreflections
6. Recovering Light (Inverse Light)

2. Radiometry

- Foreshortening and Solid angle
- Measuring light : radiance
- Light at surface : interaction between light and surface
  - irradiance = light arriving at surface
  - BRDF
  - outgoing radiance

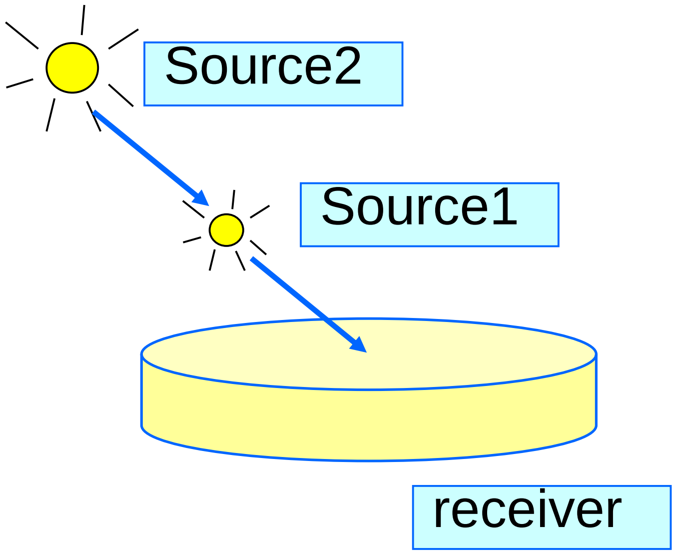


- Special cases and simplifications : Lambertain, specular, parametric and non-parametric models

Geometry and Foreshortening

Two sources that look the same to a receiver must have same effect on the receiver;

Two receivers that look the same to a source must receive the same energy.

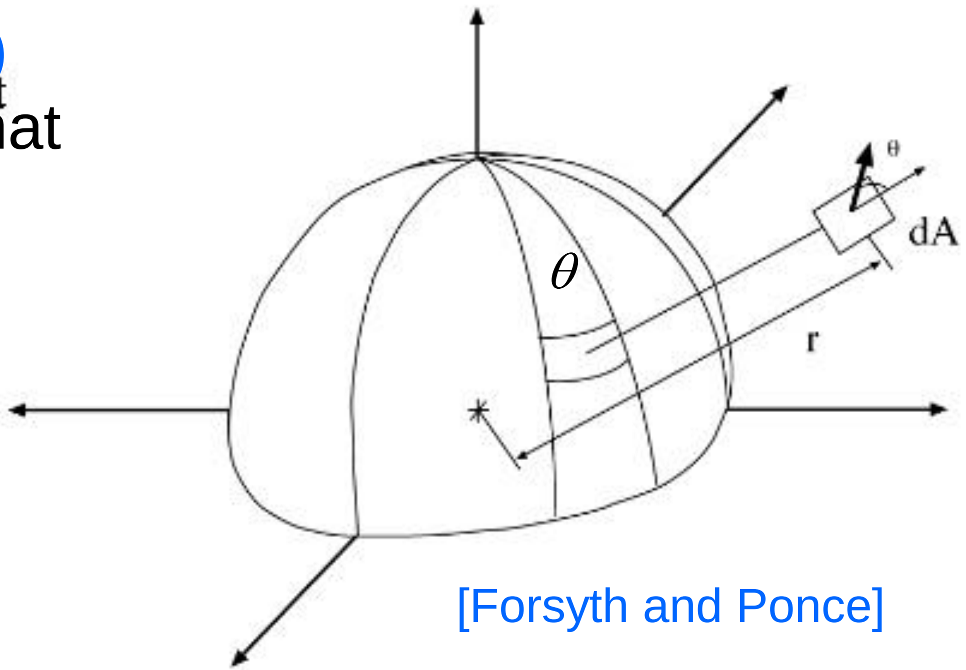


Solid angle

The solid angle subtended by a region at a point is the area projected on a unit sphere centered at the point

- Measured in steradians (sr)
- Foreshortening : patches that look the same, same solid angle.

$$d\omega = \frac{dA \cos\theta_n}{r^2}$$



[Forsyth and Ponce]

Integration inf in spherical coord:

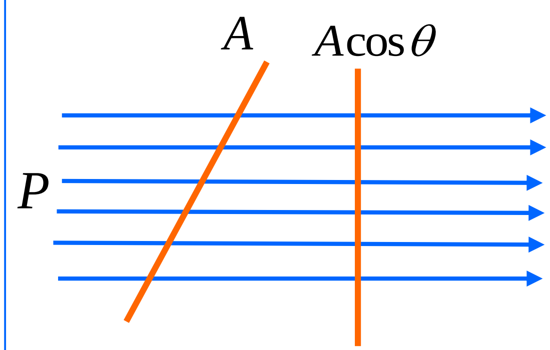
$$d\omega = \sin\theta \, d\theta \, d\phi$$

Radiance – emitted light

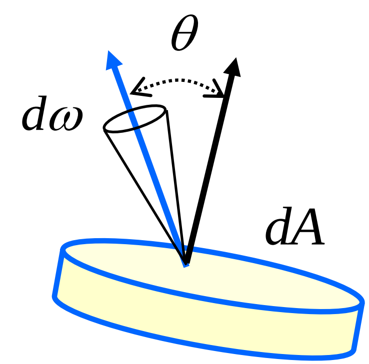
Radiance = power traveling at some point in a direction per unit area perp to direction of travel, per solid angle

- unit = watts/(m^2sr)
- constant along a ray

$$L(\mathbf{x},\theta,\phi) = \frac{P}{(dA\cos\theta)d\omega}$$



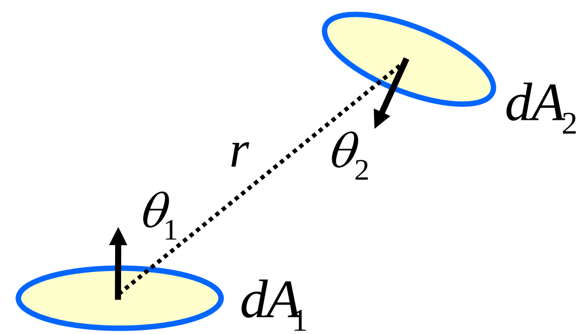
Forshortening



Radiance transfer :

Power received at dA_2 at dist r from emitting area dA_1

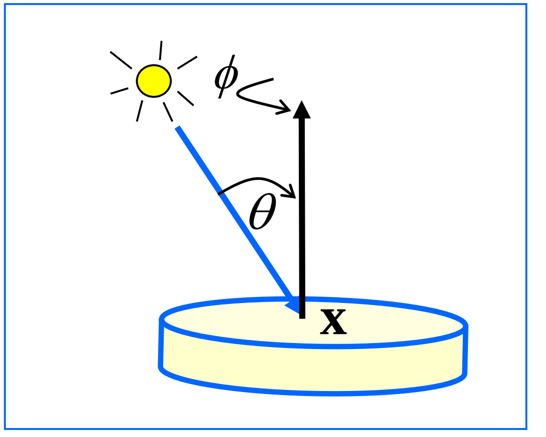
$$P_{1\to2} = L\,dA_1\cos\theta_1 \underbrace{\boxed{\left(\frac{dA_2\cos\theta_2}{r^2}\right)}}_{d\omega_{21}} \qquad P_{1\to2} = P_{2\to1}$$



Light at surface : irradiance

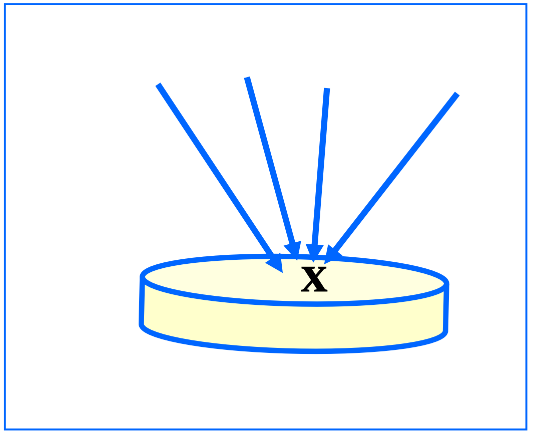
Irradiance = unit for light arriving at the surface

$$dE(\mathbf{x}) = L(\mathbf{x},\theta,\phi)\cos\theta d\omega$$

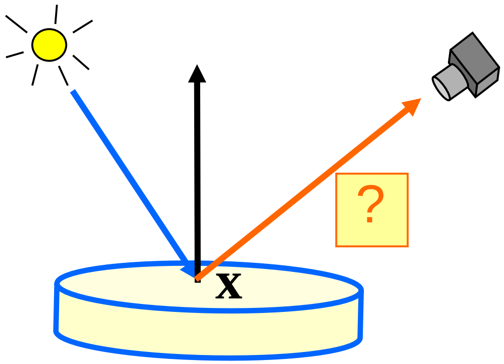


Total power = integrate irradiance over all incoming angles

$$E(\mathbf{x}) = \int_0^{2\pi}\int_0^{\pi/2} L(\mathbf{x},\theta,\phi)\cos\theta \underbrace{\boxed{\sin\theta d\theta d\phi}}_{d\omega}$$



Light leaving the surface and BRDF



many effects :

- transmitted - glass
- reflected - mirror
- scattered – marble, skin
- travel along a surface, leave some other
- absorbed - sweaty skin

Assume:

- surfaces don't fluorescent
- cool surfaces
- light leaving a surface due to light arriving

BRDF = Bi-directional reflectance distribution function

Measures, for a given wavelength, the fraction of incoming irradiance from a direction ω_i in the outgoing direction ω_o [Nicodemus 70]

$$\rho(\mathbf{x},\theta_i,\phi_i,\theta_o,\phi_o) = \frac{L_o(\mathbf{x},\theta_o,\phi_o)}{L_i(\mathbf{x},\theta_i,\phi_i)\cos\theta_i d\omega}$$

Reflectance equation : measured radiance

(radiosity = power/unit area leaving surface

$$L_o(\mathbf{x},\theta_o,\phi_o) = \int_\Omega \rho(\mathbf{x},\theta_i,\phi_i,\theta_o,\phi_o) L(\theta_i,\phi_i)\cos(\theta_i) d\omega_i$$

Radiosity - summary

Radiance	Light energy along a ray	$L(\theta,\phi) = \dfrac{P}{(dA\cos\theta)d\omega}$
Irradiance	Unit incoming light	$dE(\mathbf{x}) = L(\mathbf{x},\theta,\phi)\cos\theta d\omega$
Total Energy incoming	Energy at surface	$E_i(\mathbf{x}) = \int_{\omega} L(\mathbf{x},\theta,\phi)\cos\theta d\omega$
Radiosity	Unit outgoing radiance	$L_o(\mathbf{x},\theta_o,\phi_o) = \int_{\Omega} \rho(\mathbf{x},\theta_i,\phi_i,\theta_o,\phi_o) L(\theta_i,\phi_i)\cos(\theta_i) d\omega_i$
Total energy leaving	Energy leaving the surface	$E_o = \int_{\Omega_o} \left[\int_{\Omega_i} \rho(\mathbf{x},\theta_i,\phi_i,\theta_o,\phi_o) L(\theta_i,\phi_i)\cos(\theta_i) d\omega_i \right] \cos(\theta_o) d\omega_o$

Example: Sunlight 1kW/m^2 . Artificial light <1/10th

3. BRDF properties

BRDF = Bi-directional reflectance distribution function

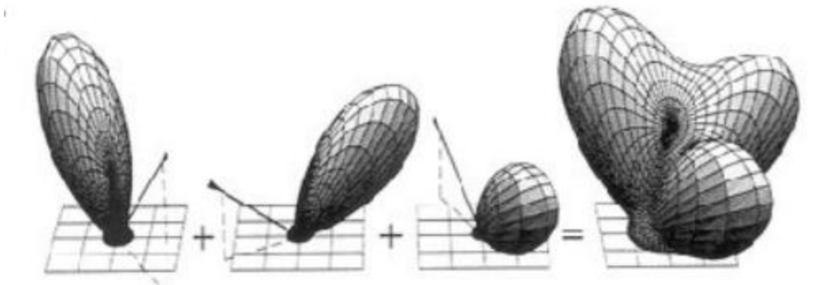
Measures, for a given wavelength, the fraction of incoming irradiance from a direction ω_i in the outgoing direction ω_o [Nicodemus 70]

Properties :

- Non-negative
- Helmholtz reciprocity
- Linear

$$\rho(\theta_i, \phi_i, \theta_o, \phi_o) \geq 0$$

$$\rho(\theta_i, \phi_i, \theta_o, \phi_o) = \rho(\theta_o, \phi_o, \theta_i, \phi_i)$$



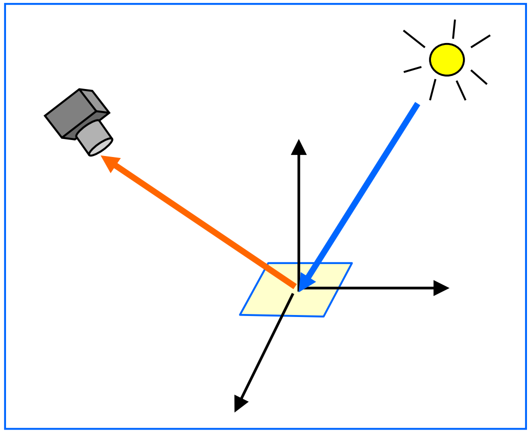
From Sillion, Arvo, Westin, Greenberg

- Total energy leaving a surface less than total energy arriving at surface

$$\int_{\Omega_i} L(\theta_i, \phi_i)\cos(\theta_i)d\omega_i \geq \int_{\Omega_o}\left[\int_{\Omega_i}\rho(\mathbf{x}, \theta_i, \phi_i, \theta_o, \phi_o)L(\theta_i, \phi_i)\cos(\theta_i)d\omega_i\right]\cos(\theta_o)d\omega_o$$

BRDF properties

isotropic (3DOF)



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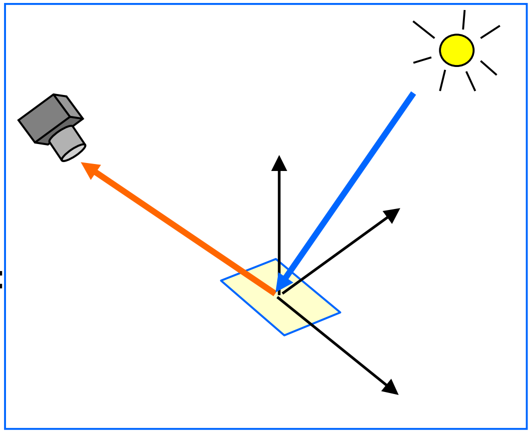
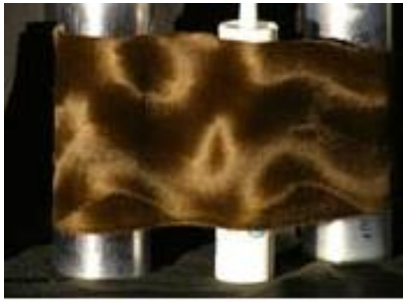


$$\rho(\theta_i,\phi_i,\theta_o,\phi_o) = \rho(\theta_i,\theta_o,\phi_i-\phi_o)$$

anisotropic (4 DOF)



[Hertzmann&Seitz CVPR03]

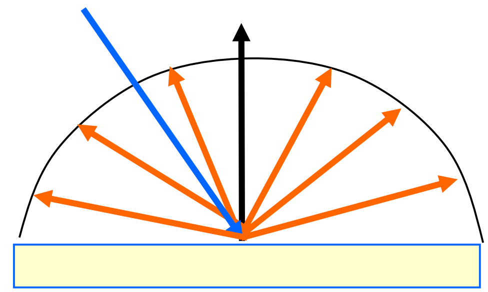
Lambertian BRDF

- Emitted radiance constant/equal in all directions
- Models – perfect diffuse surfaces : clay, mate paper, …
- BRDF = constant = albedo
- One light source = dot product normal and light direction

$$L_o(\mathbf{x}) = \rho L_i(\mathbf{x}, \theta_i, \phi_i) \cos\theta_{\theta_i,\phi}$$

$$= \rho(\mathbf{N} \bullet \mathbf{L}_i)$$

albedo normal light dir



Diffuse reflectance acts like a low pass filter on the incident illumination.

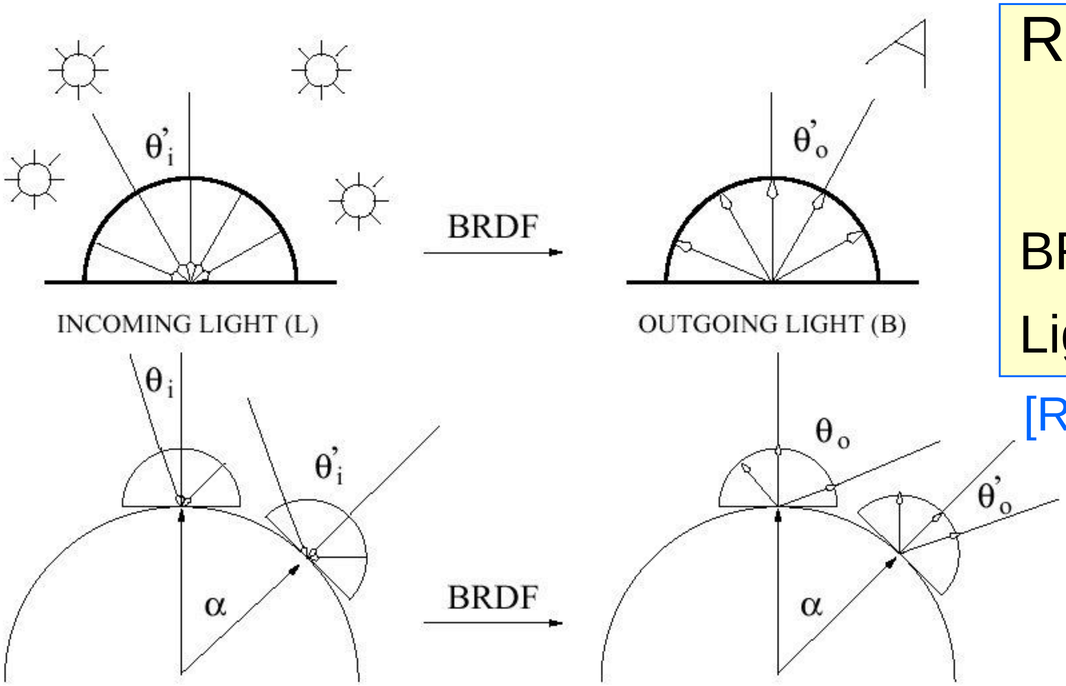
$$L_o(\mathbf{x}, \theta_o, \phi_o) = \int_{\Omega'} \rho L(\theta_i, \phi_i) \cos(\theta_i) d\omega_i$$

Reflection as convolution

Reflectance equation

$$L_o(\mathbf{x},\theta_o,\phi_o) = \int_{\Omega'} \rho(\mathbf{x},\theta_i',\phi_i',\theta_o',\phi_o')L(\theta_i,\phi_i)\cos(\theta_i)d\omega_i$$

$$= \int_{\Omega} \rho(\mathbf{x},\theta_i',\phi_i',\theta_o',\phi_o')L(R_{\alpha,\beta}(\theta_i',\phi_i'))\cos(\theta_i)d\omega_i$$



Reflection behaves like a convolution in the angular domain

BRDF – filter

Light - signal

[Ramamoorthi and Hanharan]

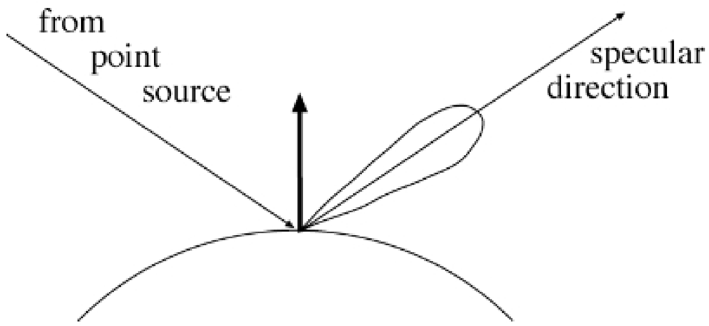
Specular reflection

Smooth specular surfaces

- Mirror like surfaces
- Light reflected along specular direction
- Some part absorbed

Rough specular surfaces

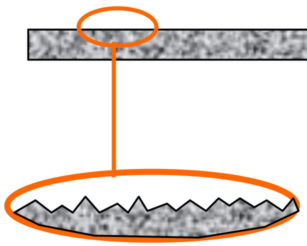
- Lobe of directions around the specular direction
- Microfacets



Lobe

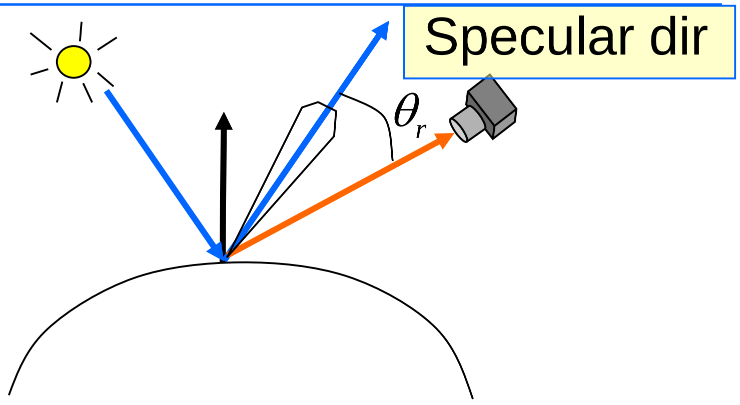
- Very small – mirror
- Small – blurry mirror
- Bigger – see only light sources
- Very big – fait specularities

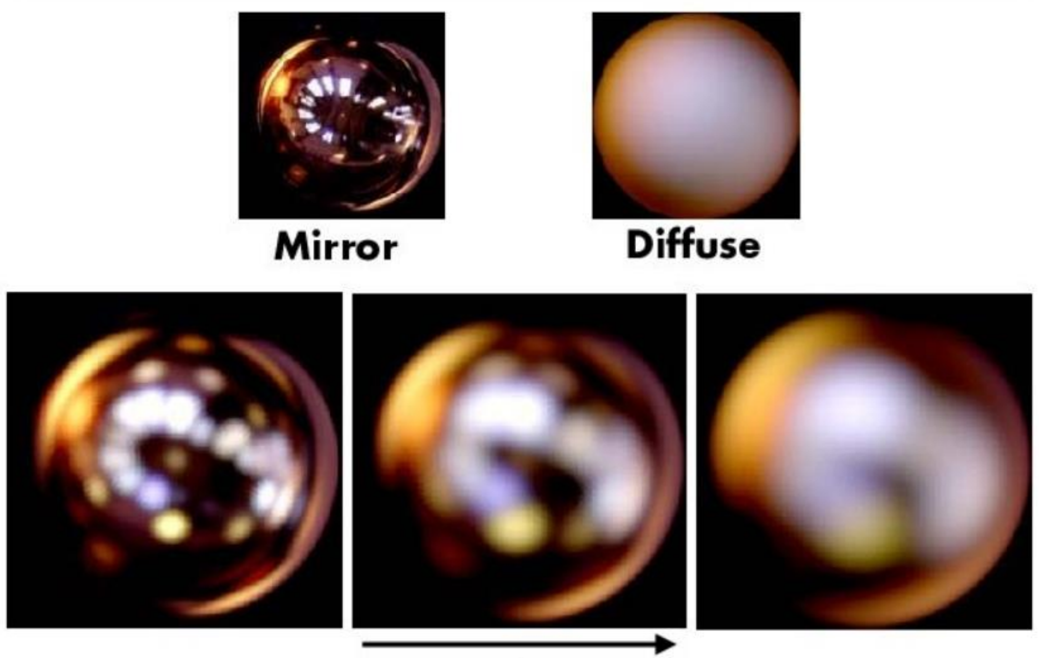
Phong model



Symmetric V shaped microfacets

$$\rho_{Phong} = k_d + k_s \frac{(\cos\theta_r)^n}{\cos\theta_i}$$





Modeling BRDF

Parametric model:

- Lambertian, Phong
- Physically based:
  - Specular [Blinn 1977] [Cook-Torrace 1982][Ward 1992]
  - Diffuse [Hanharan, Kreuger 1993]
  - Generalized Lambertian [Oren, Nayar 1995]
  - Throughly Pitted surfaces [Koenderink et al 1999]
- Phenomenological:
  - [Koenderink, Van Doorn 1996]

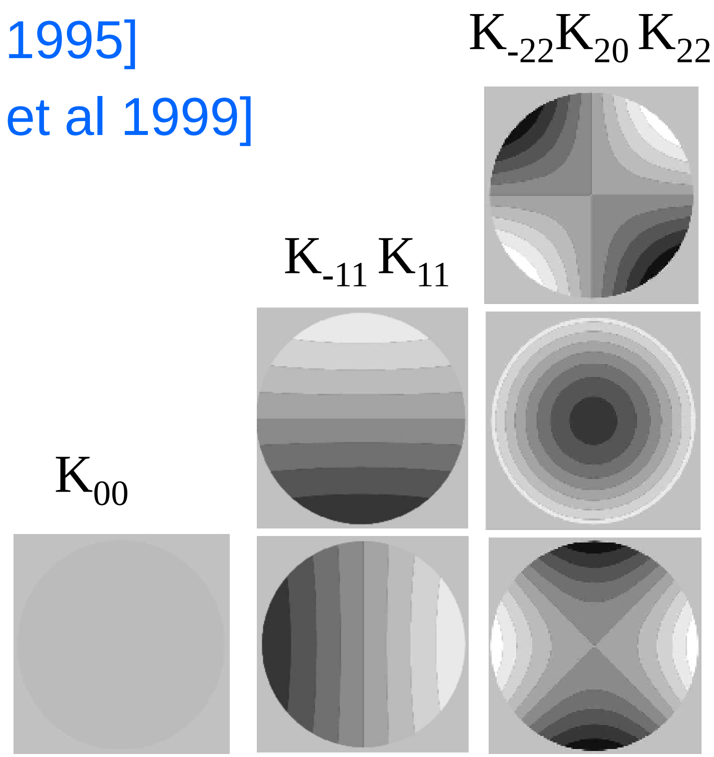
    summarize empirical data

    orthonormal functions on the $\mathbf{H}_{S^2} \times \mathbf{H}_{S^2}$

    ($\mathbf{H}_{S^2}$ hemisphere)

    same topol. as unit disk

    (Zernike Polynomials)

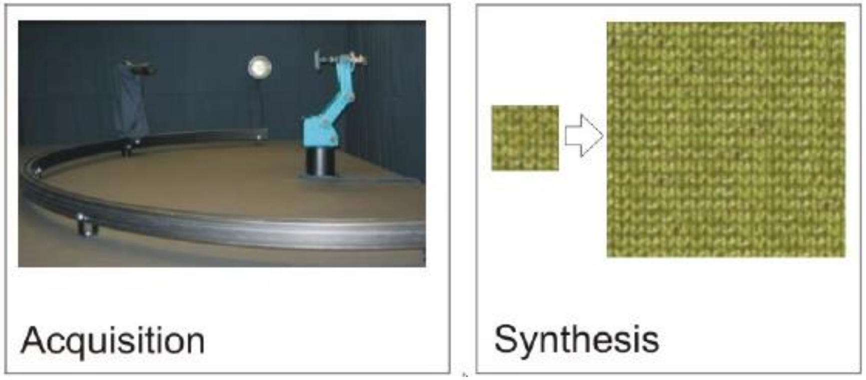


Measuring BRDF

Gonioreflectometers

- Anisotropic 4 DOF
- Non-uniform

BTF [Dana et al 1999]



[Müller 04]

More than BRDF – BSSRDF

(bidirectional surface scattering distribution)

BRDF

BSSRDF

[Jensen, Marschner, Leveoy, Hanharan 01]

Do SFS from here.

4. Light representations

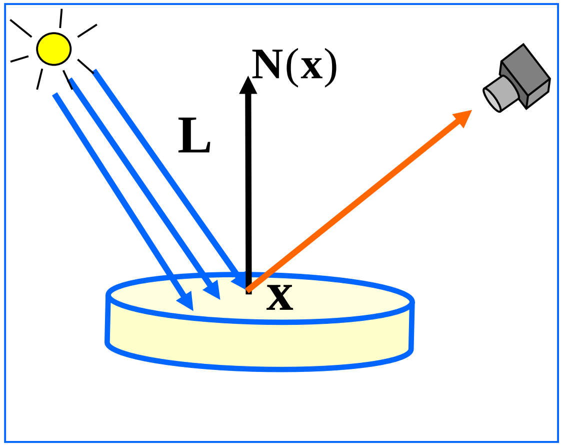
Light source –

theoretical framework [Langer, Zucker-What is a light source]

Point light sources

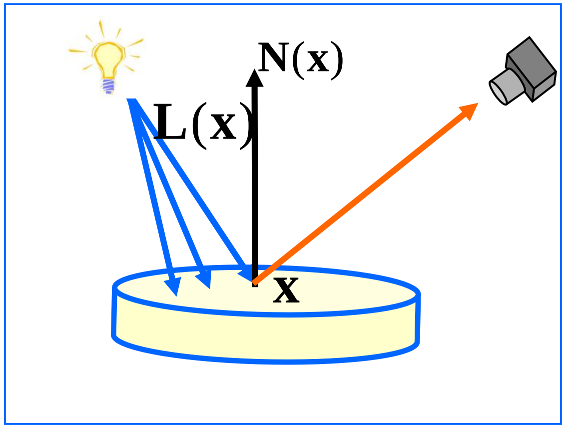
- <u>Infinite</u>

$$L_o(\mathbf{x}) = \rho(\mathbf{x}) E \cos\theta_i = \rho(\mathbf{x}) \mathbf{N}(\mathbf{x}) \bullet \mathbf{L}$$



- <u>Nearby</u>

$$L_o(\mathbf{x}) = \rho(\mathbf{x}) \frac{E \cos\theta_i(\mathbf{x})}{r^2} = \rho(\mathbf{x}) \frac{\mathbf{N}(\mathbf{x}) \bullet \mathbf{L}(\mathbf{x})}{r^2}$$



<u>Choosing a model</u>

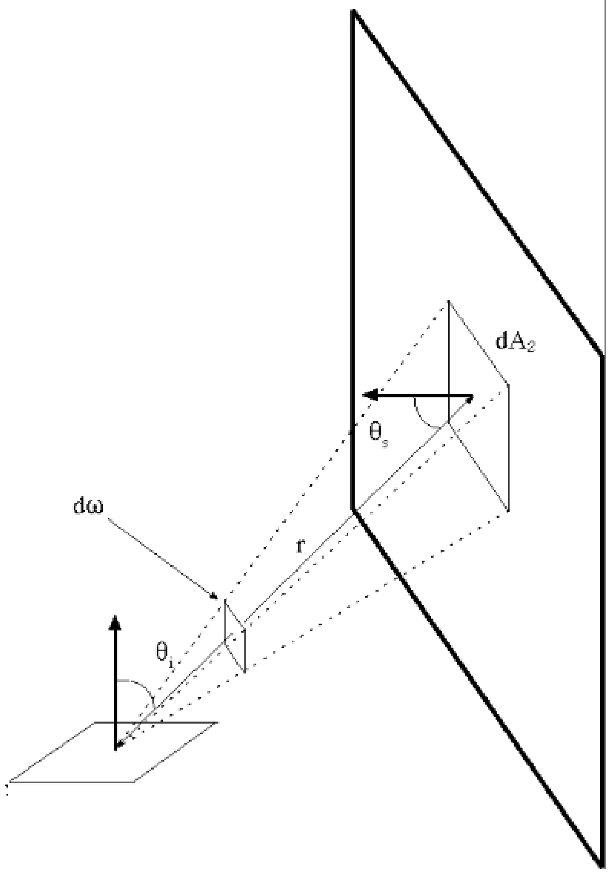
- infinite - sun
- finite - distance to source is similar in magnitude with object size and distance between objects
  - indoor lights

Area sources

Examples : white walls, diffuse boxes

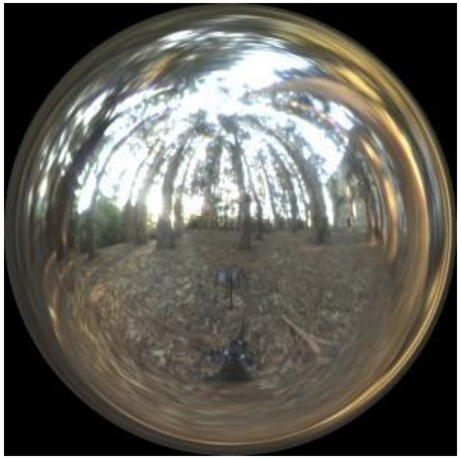
Radiosity : adding up contributions over the section of the view hemisphere subtended by the source

$$L_o(x) = \rho(x) \int_{source} E(Q) \frac{\cos\theta_i \cos\theta_s}{\pi r^2} dA_Q$$

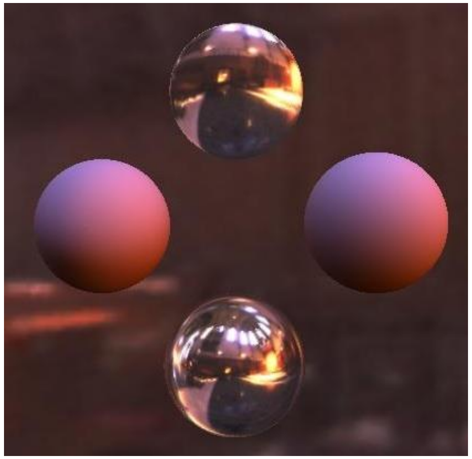


Enviromental map

Illumination hemisphere
Large number of infinite point light sources



[Debevec]



5. Image cues

shading, shadows, specularities …

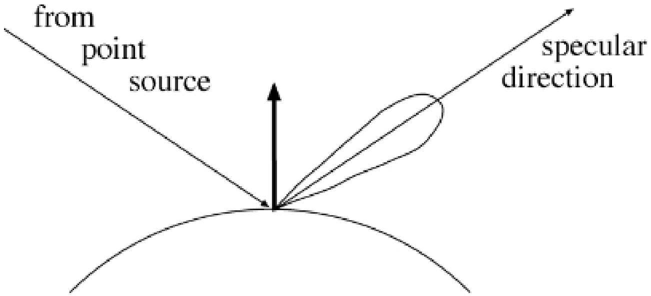
Shading

Lambertian reflectance $$L_o(\mathbf{x}) = \rho L \boxed{\cos\theta} = \rho L \boxed{(\mathbf{N} \bullet \mathbf{L}_i)}$$

Shading = observed <u>smooth</u> color variation due to <u>Lambertian</u> reflectance

Specular highlights

High frequency changes in observed radiance due to general BRDF (shiny material)



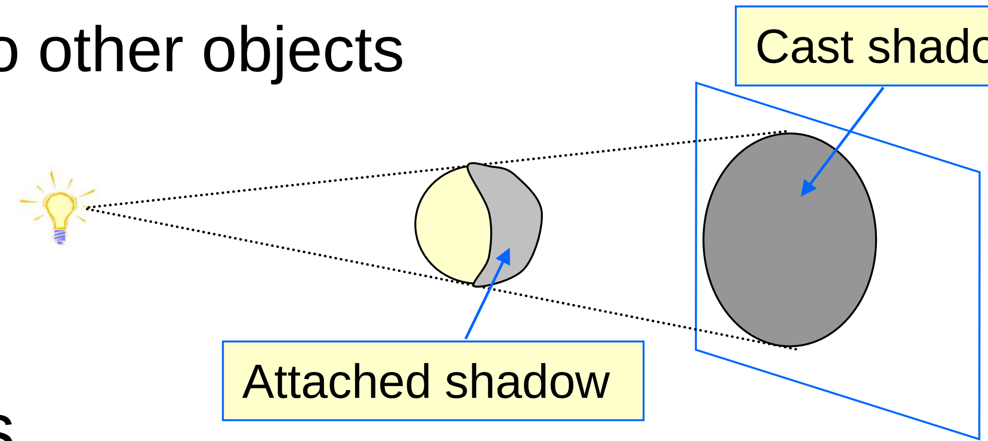
Shadows (local)

1. <u>Point light sources</u>

   Points that cannot see the source – modeled by a visibility binary value

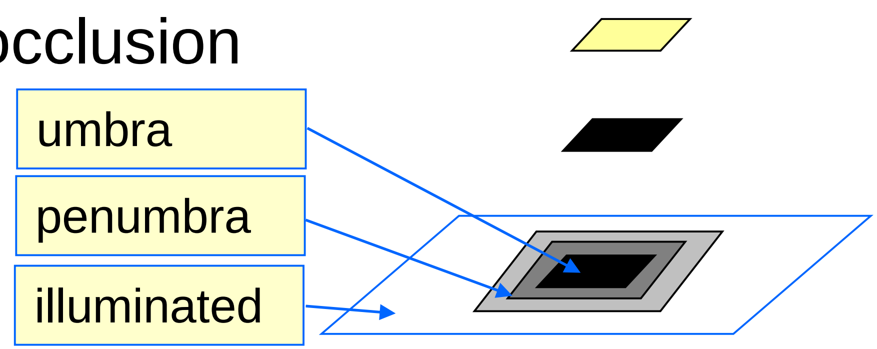
   attached shadows = due to object geometry (self-shadows)

   cast shadows = due to other objects



2. <u>Area light sources</u>

   Soft shadows – partial occlusion



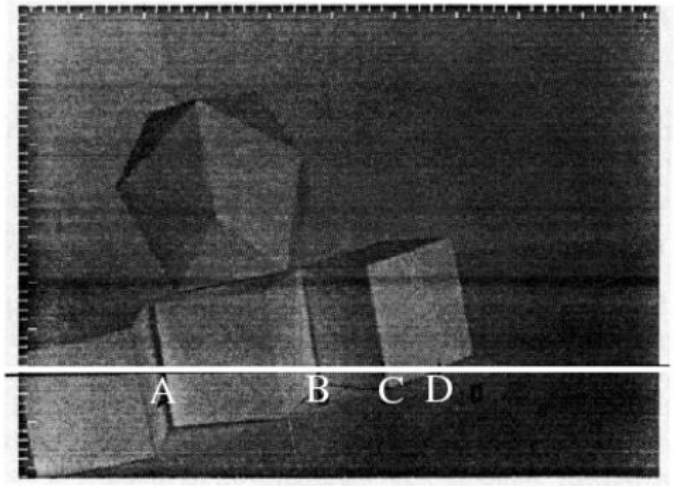
Interreflections

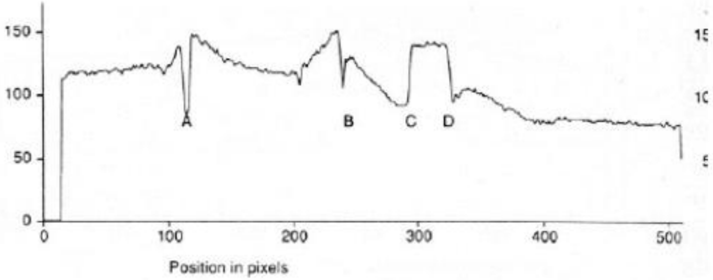
Local shading – radiosity only due to light sources

[computer vision, real-time graphics]

Global illumination – radiosity due to radiance reflected from light sources as well as surfaces

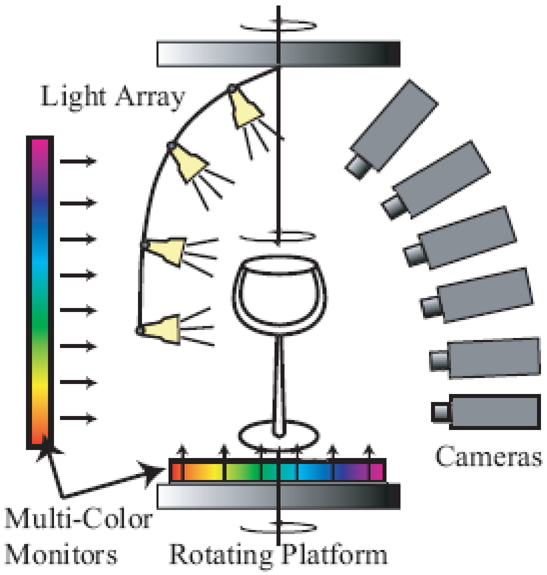
[computer graphics]





White room under bright light. Below cross-section of image intensity
[Forsyth, Zisserman CVPR89]

Transparency



Special setups for image aquisition

Enviroment mating
[Matusik et al Eurographics 2002]
[Szeliski et al Siggraph 2000]

6. Inverse light

$$L_o(\mathbf{x},\theta_o,\phi_o) = \int_{\Omega'} \rho(\mathbf{x},\theta_i',\phi_i',\theta_o',\phi_o') \boxed{L(\theta_i,\phi_i)} \cos(\theta_i) d\omega_i$$

Deconvolution of light from observed radiance

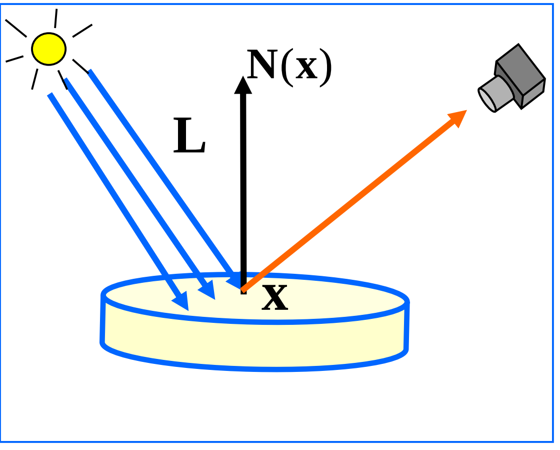
Assumptions:

- known camera position
- known object geometry
- [known or constant BRDF]
- [uniform or given texture]

Estimating multiple point light sources

Estimating complex light : light basis

Estimating point light sources



Lambertian reflectance – light from shading

Infinite single light source

$$L_o(\mathbf{x}) = \rho(\mathbf{x}) L \cos\theta \;=\rho L(\mathbf{N}(\mathbf{x}) \bullet \mathbf{L})$$

- known or constant albedo ρ
- known $\mathbf{N}(\mathbf{x})$
- recover L (light color) and $\mathbf{L}$ (direction) from >= 4 points.

Multiple light sources

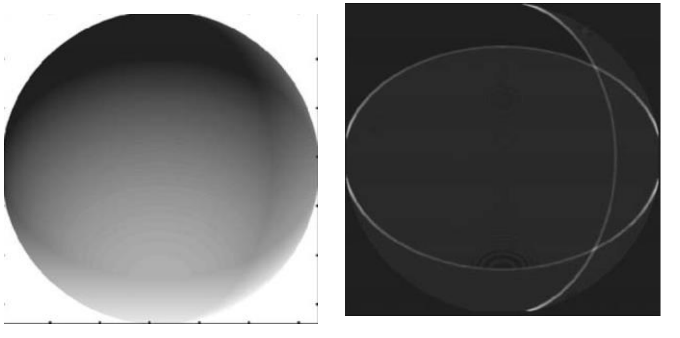
Calibration sphere

Critical points/curves

- Sensitive to noise

[Yang Yuille 91]

[Bouganis 03]



Estimating complex light

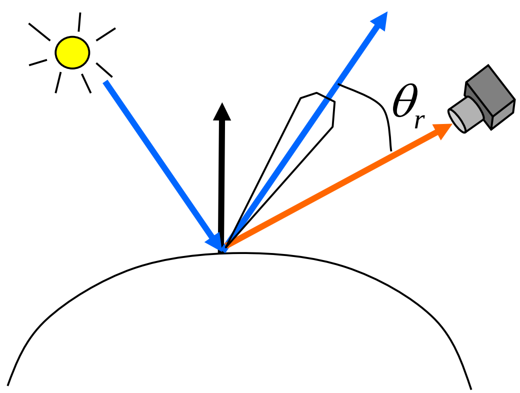
Diffuse reflectance acts like a low pass filter on the incident illumination.

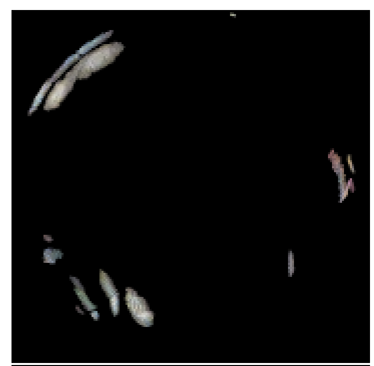
Can only recover low frequency components. Use other image cues !

Light from specular reflections

- Recover a discrete illumination hemisphere
- Specular highlights appear approximately at mirror directions between light and camera rays
- Trace back and compute intersection with hemisphere

Specular dir





Recovered hemisphere

Capture light direcly using a mirror sphere

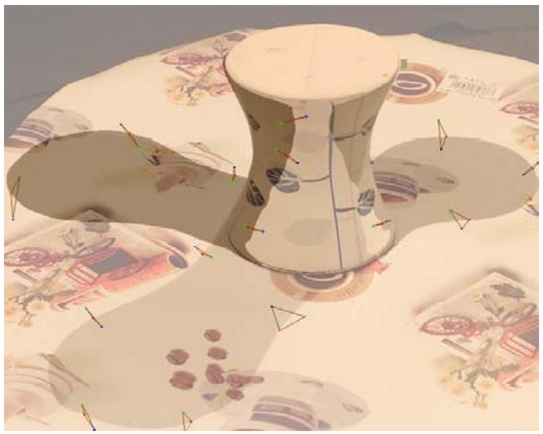
[Nishimo,Ikeuchi ICCV 2001]

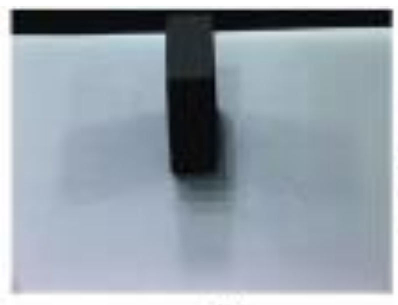
Estimating complex light

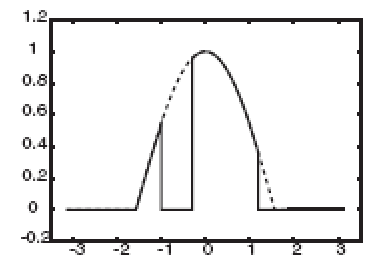
Light from cast shadows

[Li Lin Shun 03]

[Sato 03]







- Shadows are caused by light being occluded by the scene.
- The measured radiance has high frequency components introduced by the shadows.

$$L_o(\mathbf{x},\theta_o,\phi_o)=\int_{\Omega}\boxed{V(\mathbf{x},\theta_i,\phi)}\rho L(\theta_i,\phi_i)\cos(\theta_i)d\omega_i$$

Shadow indicator

Light basis representation

Spherical harmonics basis

- Analog on the sphere to the Fourier basis on the line or circle
- Angular portion of the solution to Laplace equation in spherical coordinates
- Orthonormal basis for the set of all functions on the surface of the sphere $\nabla^2 \psi = 0$

$$Y_{lm}(\theta, \phi) = \underbrace{\sqrt{\frac{(2l+1)}{4\pi} \frac{(l-|m|)!}{(l+|m|)!}}}_{\text{Normalization factor}} \underbrace{P_{l|m|}(\cos\theta)}_{\text{Legendre functions}} \underbrace{e^{im\phi}}_{\text{Fourier basis}}$$

Illustration of SH

m

l

0

1

2

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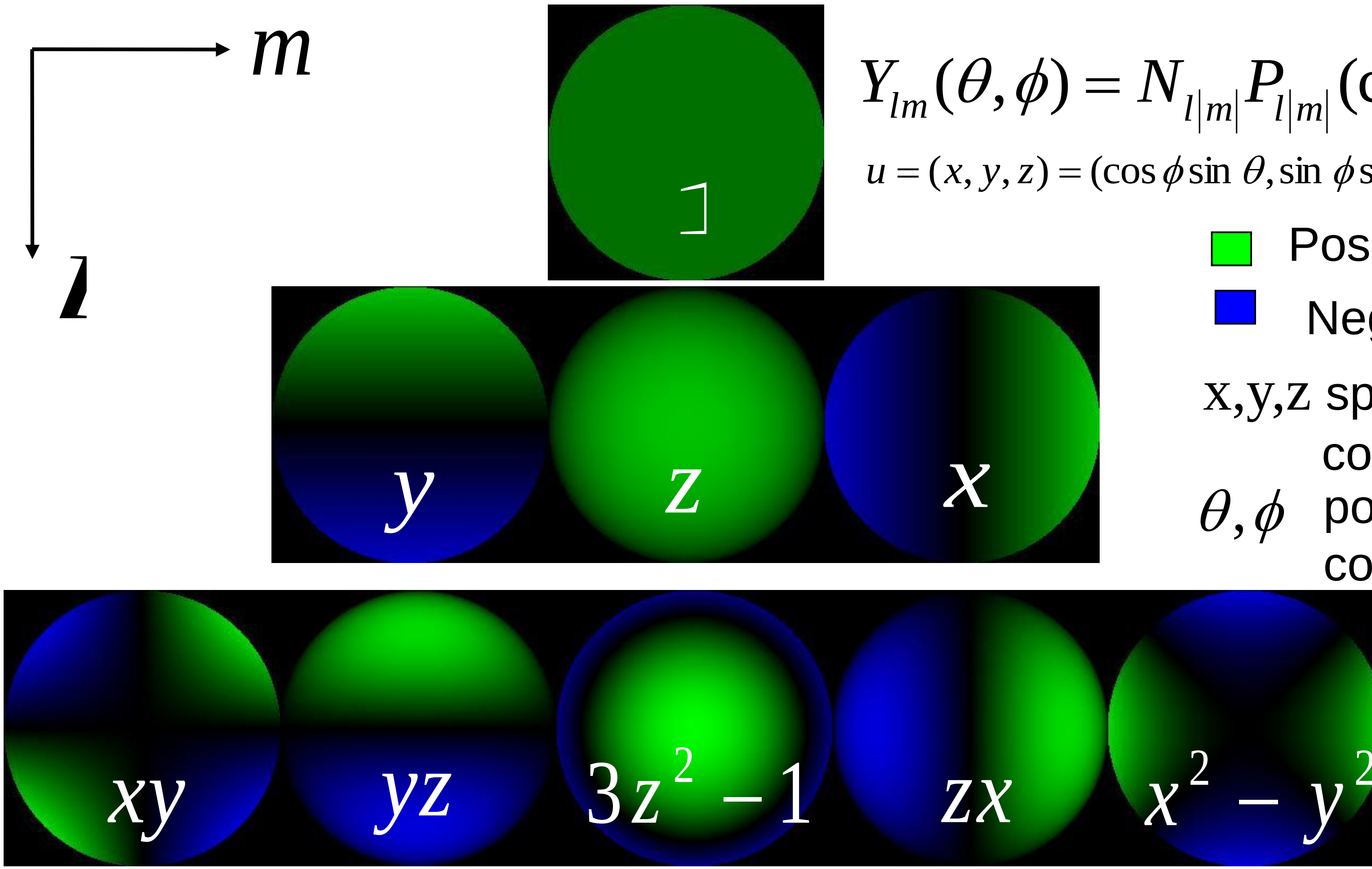


-2 -1 0 1 2

$$Y_{lm}(\theta,\phi)=N_{l|m|}P_{l|m|}(\cos\theta)e^{im\phi}$$

$$u=(x,y,z)=(\cos\phi\sin\theta,\sin\phi\sin\theta,\cos\theta)$$

Positive

Negative

x,y,z space coordinates;

θ,ϕ polar coordinates

odd components $\quad Y_{lm}=Y_{l|m|}^{e}\pm iY_{l|m|}^{o}\quad$ even components

Properties of SH

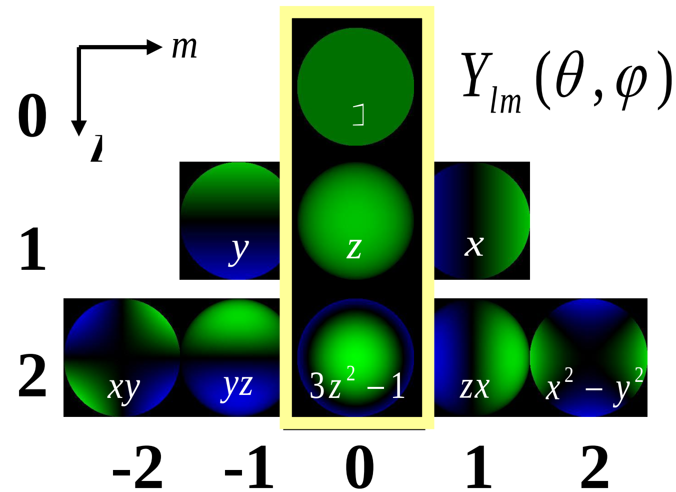
Function decomposition

f piecewise continuous function on the surface of the sphere

$$f(u) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} f_{lm} Y_{lm}(u)$$

where

$$f_{lm} = \int_{S^2} f(u) Y^{*}_{lm}(u) du$$



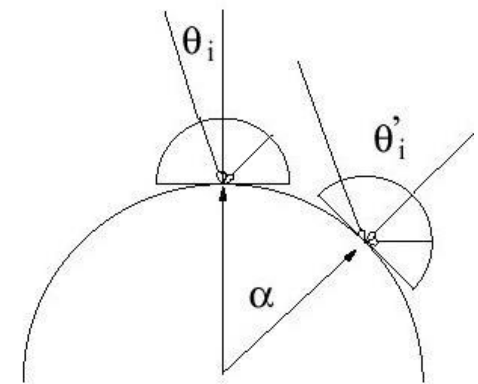
Rotational convolution on the sphere

Funk-Hecke theorem:

k circularly symmetric bounded integrable function on [-1,1] $\quad k(u) = \sum_{l=0}^{\infty} k_l Y_{l0}$

$$k * Y_{lm} = \alpha_l Y_{lm} \qquad \alpha_l = \sqrt{\frac{4\pi}{2l+1}} k_l$$

convolution of a (circularly symmetric) function k with a spherical harmonic Y_{lm} results in the same harmonic, scaled by a scalar α_l.



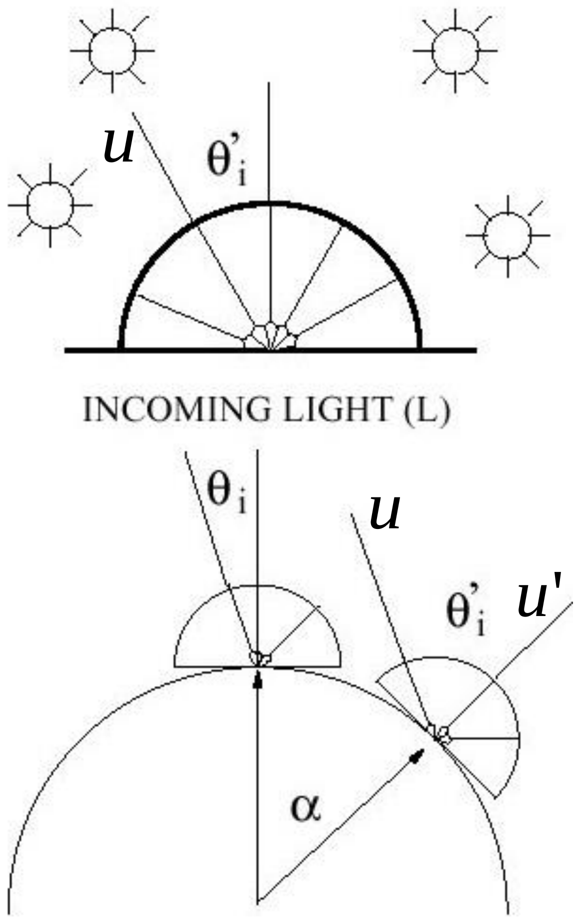
Reflectance as convolution

Lambertian reflectance

One light $$R(u') = l(u)\rho \max(0, u \bullet u')$$

Lambertian kernel $$k(u \bullet u') = \max(0, u \bullet u')$$

Integrated light $$R(u') = \int_{S^2} k(u \bullet u') l(u) du$$



SH representation

light

$$l(u) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} l_{lm} Y_{lm}(u)$$

Lambertian kernel

$$k = \sum_{l=0}^{\infty} k_l Y_{l0}$$

Lambertian reflectance (convolution theorem)

$$R = k * l = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left(\sqrt{\frac{4\pi}{2l+1}} k_l l_{lm}\right) Y_{lm} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} r_{lm} Y_{lm}$$

Convolution kernel

Lambertian kernel

$$k(u \bullet u') = \max(0, u \bullet u')$$

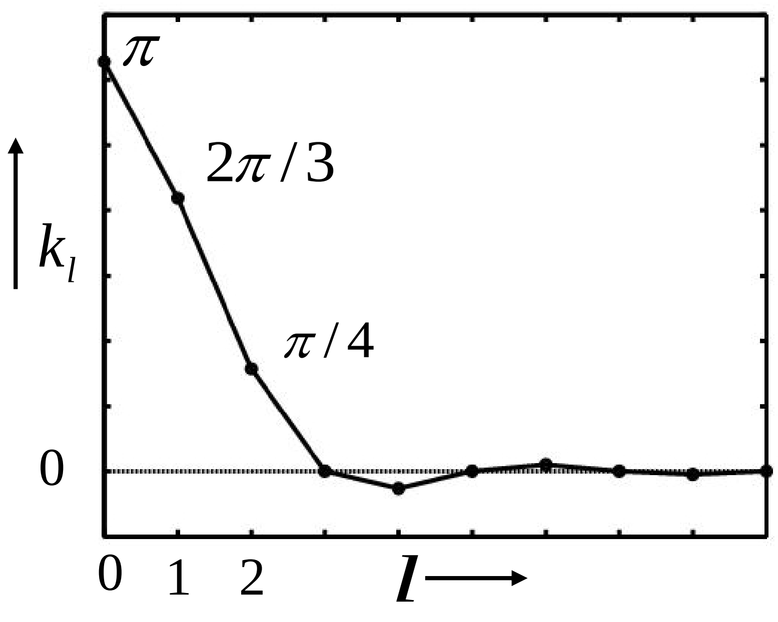
$$k = \sum_{l=0}^{\infty} k_l Y_{l0}$$

$$k_l = \begin{cases} \dfrac{\sqrt{\pi}}{2} & n = 0 \\ \dfrac{\sqrt{\pi}}{3} & n = 1 \\ (-1)^{l/2+1} \dfrac{\sqrt{(2l+1)\pi}}{2^l (l-1)(l+2)} \begin{pmatrix} l \\ l/2 \end{pmatrix} & n \geq 2, \text{even} \\ 0 & n \geq 2, \text{odd} \end{cases}$$

Asymptotic behavior of k_l for large l

$$k_l \approx l^{-2} \qquad r_{lm} \approx l^{-5/2}$$

- Second order approximation accounts for 99% variability
- k like a low-pass filter



[Basri & Jacobs 01]
[Ramamoorthi & Hanrahan 01]

From reflectance to images

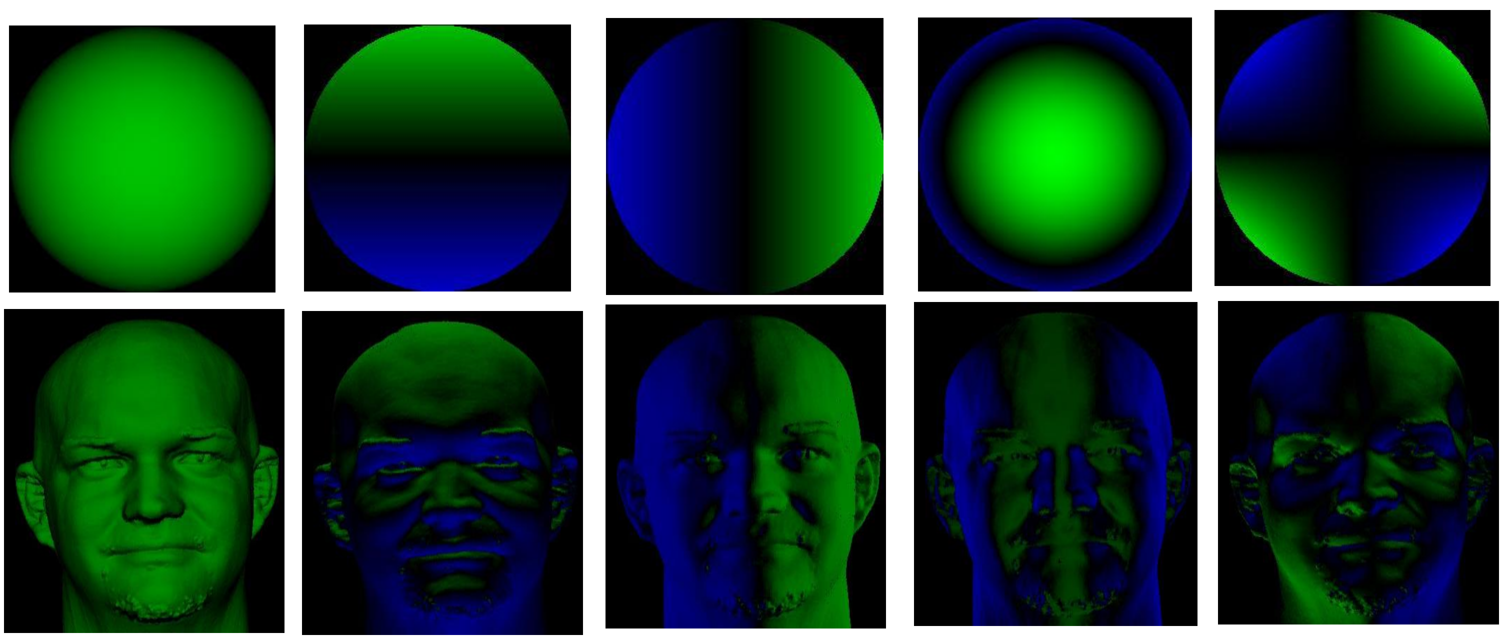
Unit sphere $\Rightarrow$ general shape
Rearrange normals on the sphere

Reflectance on a sphere

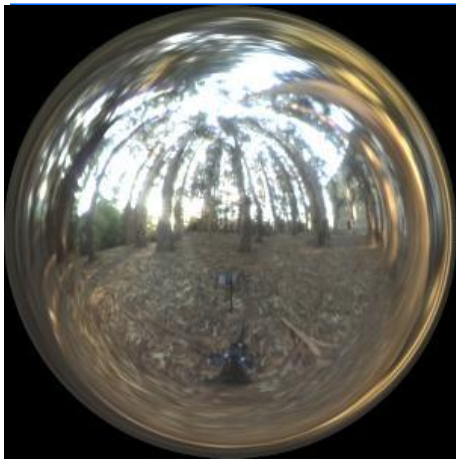
$$R = k * l = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} r_{lm} Y_{lm}$$

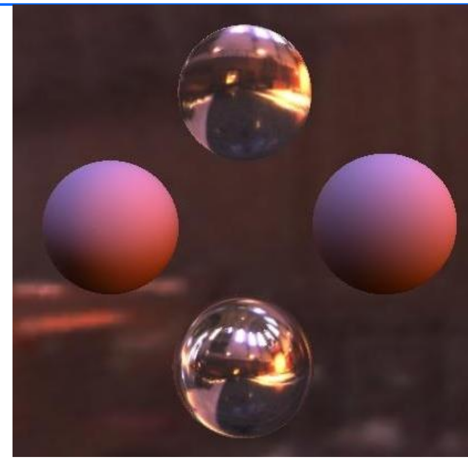
Image point with normal n_i

$$I_i = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \rho_i r_{lm} Y_{lm}(n_i)$$



Example of approximation





Exact image

9 terms approximation

Efficient rendering

- known shape
- complex illumination (compressed)

[Ramamoorthi and Hanharan: An efficient representation for irradiance enviromental map Siggraph 01]

Extensions to other basis

SH light basis limitations:

- Not good representation for high frequency (sharp) effects ! (specularities)
- Can efficiently represent illumination distribution localized in the frequency domain
- BUT a large number of basis functions are required for representing illumination localized in the angular domain.

Basis that has both frequency and spatial support

⇨ Wavelets — [Upright CRV 07] [Okabe Sato CVPR 2004]

⇨ Spherical distributions — [Hara, Ikeuchi ICCV 05]

⇨ Light probe sampling — [Debevec Siggraph 2005] [Madsen et al. Eurographics 2003]

Basis with local support

Median cut

[Debevec
Siggraph 2005]

Not localized
in frequency!

Wavelet Basis

[Upright Cobzas 07]