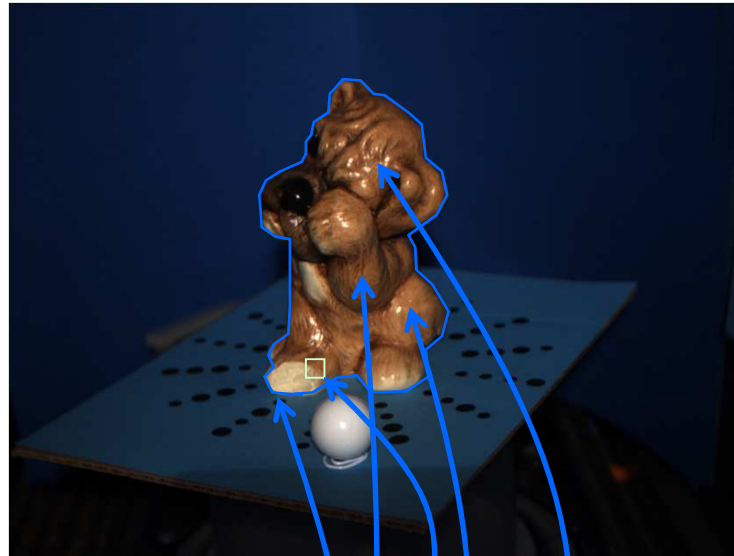


Image cues

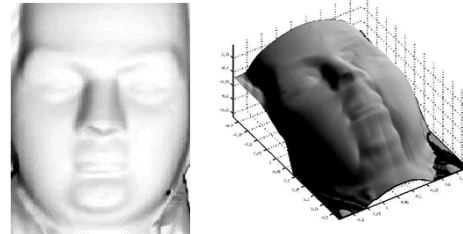


Color (texture)
Shading
Shadows
Specular highlights
Silhouette

Image cues

Shading [reconstructs normals]

shape from shading (SFS)
photometric stereo



Specular highlights

[ignore, filtered]
[parametric BRDF]



Texture [reconstructs 3D]

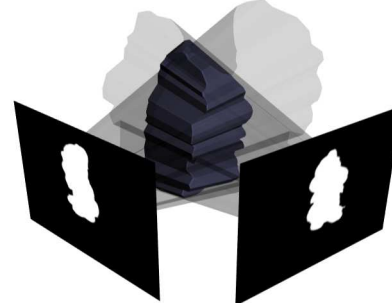
stereo (relates two views)



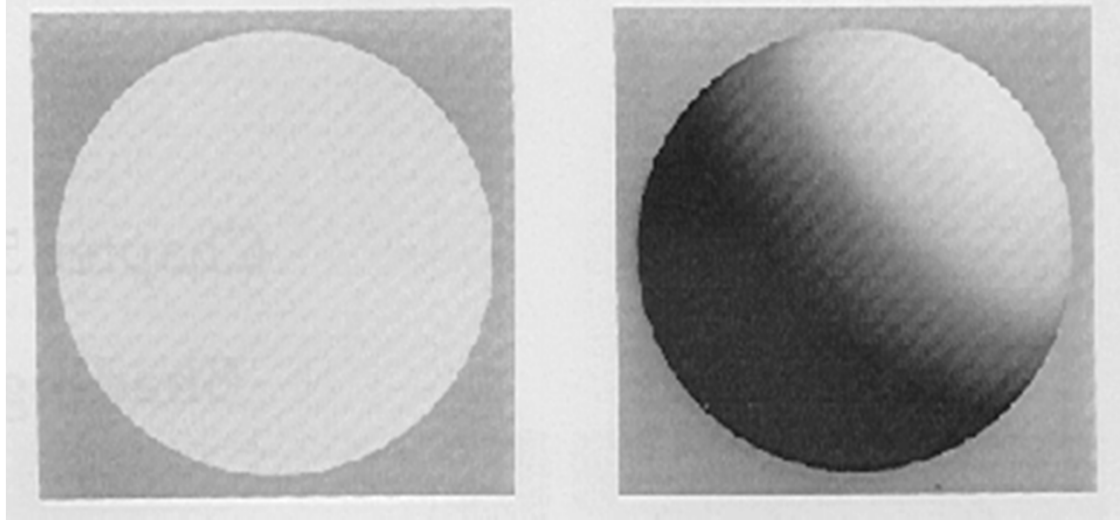
Silhouette [reconstructs 3D]

shape from silhouette

[Focus]



Geometry from shading



Shading reveals 3D shape geometry

Shape from Shading

One image

Known light direction

Known BRDF (unit albedo)

Ill-posed : additional constraints
(integrability ...)

[Horn]

Photometric Stereo

Several images, different lights

Unknown Lambertian BRDF

1. Known lights
2. Unknown lights

[Silver 80, Woodman 81]

Reconstruct normals
Integrate surface

Shading

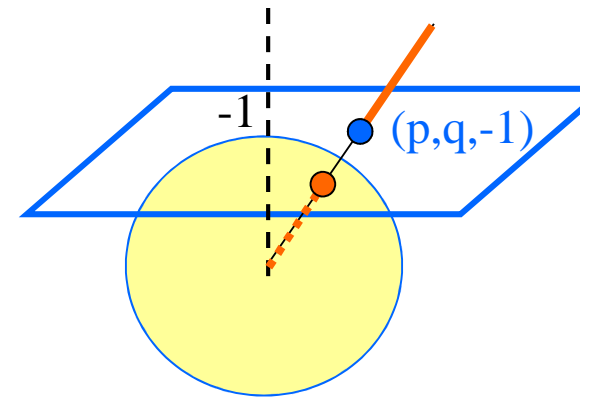
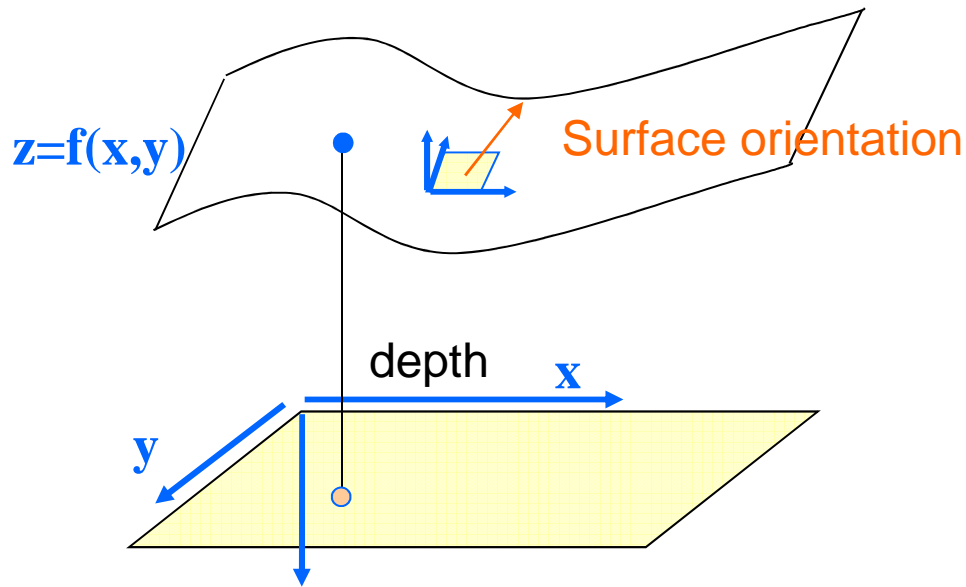
Lambertian reflectance

$$E(\mathbf{x}) = \rho L_i(\mathbf{x}, \theta_i, \phi_i) \cos \theta_i = \rho(\mathbf{n} \bullet \mathbf{l}_i)$$

The diagram illustrates the components of the Lambertian reflectance equation. Below the equation, three boxes are arranged horizontally: 'albedo' (blue border), 'normal' (yellow background, blue border), and 'light dir' (blue border). Arrows point from these boxes to the corresponding terms in the equation: 'albedo' points to ρ , 'normal' points to \mathbf{n} , and 'light dir' points to \mathbf{l}_i .

Fixing light, albedo, we can express reflectance only as function of normal.

Surface parametrization



Surface

$$s(x, y) = (x, y, f(x, y))$$

Tangent plane

$$\frac{\partial s}{\partial x} = \left(1, 0, \frac{\partial f}{\partial x}\right)^T \quad \frac{\partial s}{\partial y} = \left(0, 1, \frac{\partial f}{\partial y}\right)^T$$

Normal vector

$$\mathbf{n} = \frac{\partial s}{\partial x} \times \frac{\partial s}{\partial y} = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, -1\right)^T$$

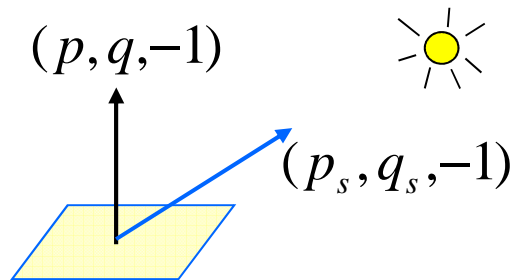
Gradient space

$$p = \frac{\partial f}{\partial x} \quad q = \frac{\partial f}{\partial y}$$

$$\mathbf{n} = (p, q, -1)$$

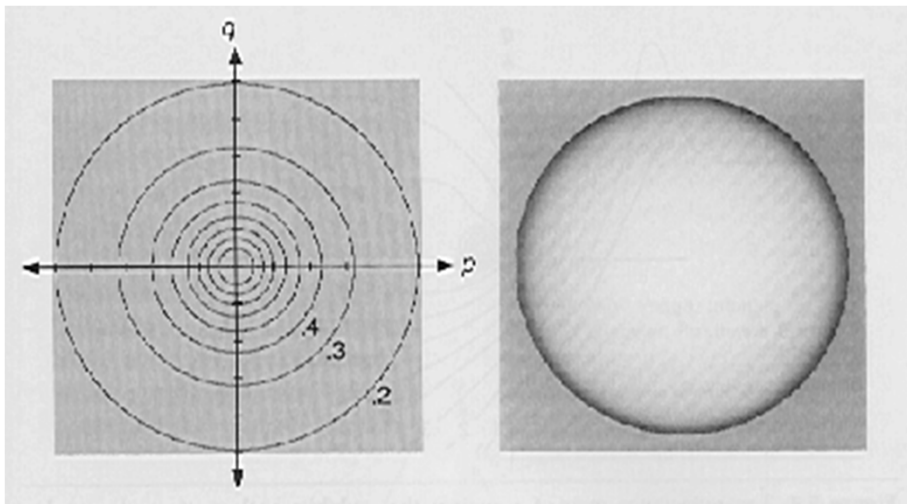
$$\hat{\mathbf{n}} = \frac{1}{\sqrt{p^2 + q^2 + 1}} (p, q, -1)$$

Lambertian reflectance map

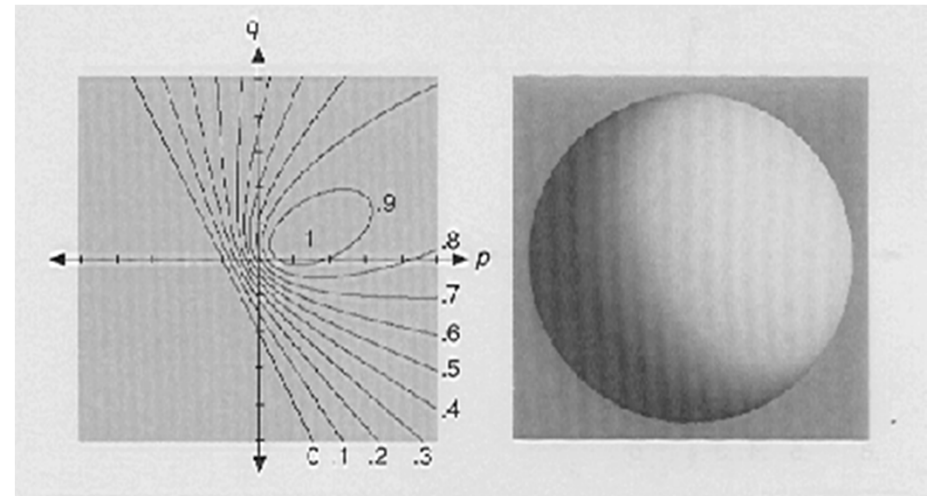


$$E(p, q) = L\rho \frac{1 + pp_s + qq_s}{\sqrt{1 + p^2 + q^2} \sqrt{1 + p_s^2 + q_s^2}}$$

Local surface orientation that produces equivalent intensities are quadratic conic sections contours in gradient space



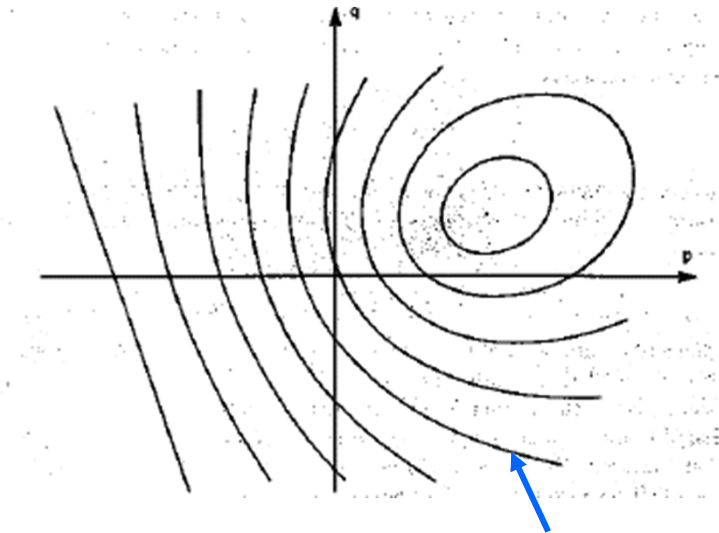
$$p_s=0, q_s=0$$



$$p_s=-2, q_s=-1$$

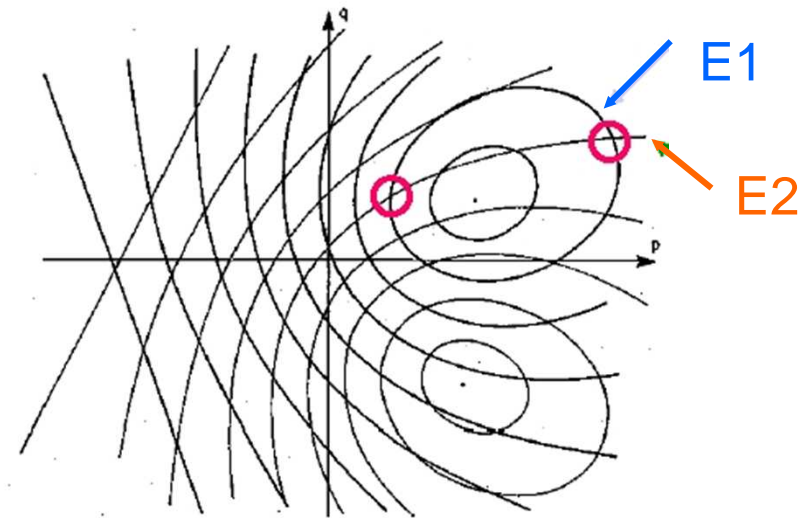
Photometric stereo

One image = one light direction



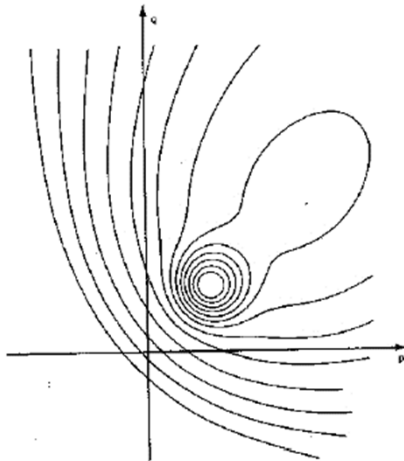
Radiance of one pixel constrains the normal to a curve

Two images = two light directions



A third image disambiguates between the two.

Normal = intersection of 3 curves



Specular reflectance

Photometric stereo



One image, one light direction

$$I(\mathbf{x}) = B(\mathbf{x}) = \rho(\mathbf{x})\mathbf{n}(\mathbf{x}) \bullet \mathbf{l}_i$$

n images, n light directions

$$\begin{bmatrix} \mathbf{l}_1^T \\ \mathbf{l}_2^T \\ \vdots \\ \mathbf{l}_n^T \end{bmatrix} \rho(\mathbf{x})\mathbf{n}(\mathbf{x}) = \begin{bmatrix} I_1^T(\mathbf{x}) \\ I_2^T(\mathbf{x}) \\ \vdots \\ I_n^T(\mathbf{x}) \end{bmatrix}; \quad \underbrace{A\rho(\mathbf{x})\mathbf{n}(\mathbf{x})}_{\mathbf{b}(\mathbf{x})} = I(\mathbf{x})$$

Given: $n \geq 3$ images with different known light dir. (infinite light)

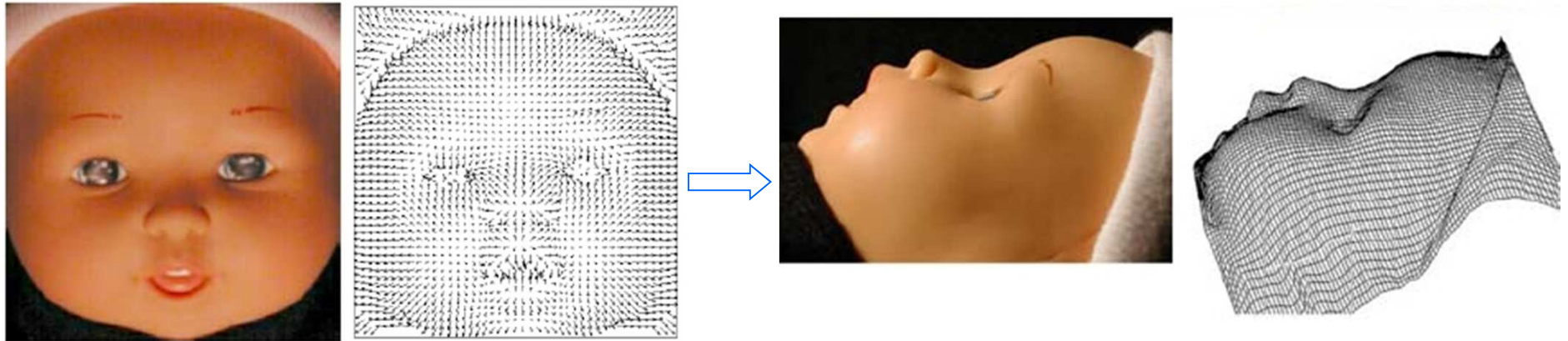
Assume: Lambertian object
orthographic camera
ignore shadows, interreflections

Recover $\mathbf{b}(\mathbf{x}) = \rho(\mathbf{x})\mathbf{n}(\mathbf{x})$

Albedo = magnitude $|\mathbf{b}(\mathbf{x})|$

Normal = normalized $\frac{\mathbf{b}(\mathbf{x})}{|\mathbf{b}(\mathbf{x})|}$

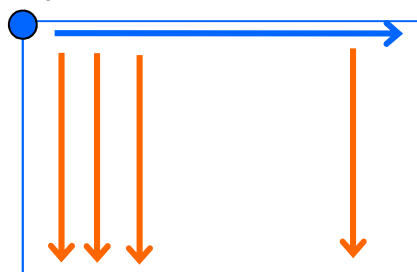
Depth from normals (1)



[D. Kriegman]

Integrate normal (gradients p, q) across the image

Simple approach – integrate along a curve from (x_0, y_0)



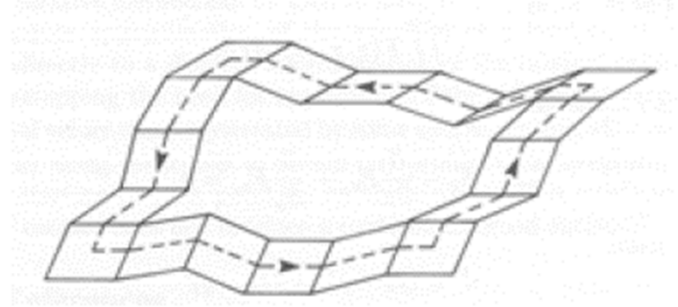
$f(x, 0)$

1. From $\mathbf{n} = (n_x, n_y, n_z)$ $p = n_x / n_z$ $q = n_y / n_z$
2. Integrate $p = \partial f / \partial x$ along $(x, 0)$ to get $f(x, 0)$
3. Integrate $q = \partial f / \partial y$ along each column

$$f(x, y) = f(x_0, y_0) + \int_{(x_0, y_0)}^{(x, y)} (p dx + q dy)$$

Depth from normals (2)

$$f(x, y) = f(x_0, y_0) + \int_{(x_0, y_0)}^{(x, y)} (pdx + qdy)$$



Integrate along a curve from (x_0, y_0)
Might not go back to the start
because of noise – depth is not
unique

Impose integrability

A normal map that produces a
unique depth map is called integrable

Enforced by $\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}; \quad \frac{\partial}{\partial y} \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \frac{\partial f}{\partial y}$



[Escher] no integrability

Impose integrability

[Horn – Robot Vision 1986]

Solve $f(x,y)$ from p,q by minimizing the cost functional

$$\iint_{\text{image}} (f_x - p)^2 + (f_y - q)^2 dx dy$$

- Iterative update using calculus of variation
- Integrability naturally satisfied
- $F(x,y)$ can be discrete or represented in terms of basis functions

Example : Fourier basis (DFT)-close form solution

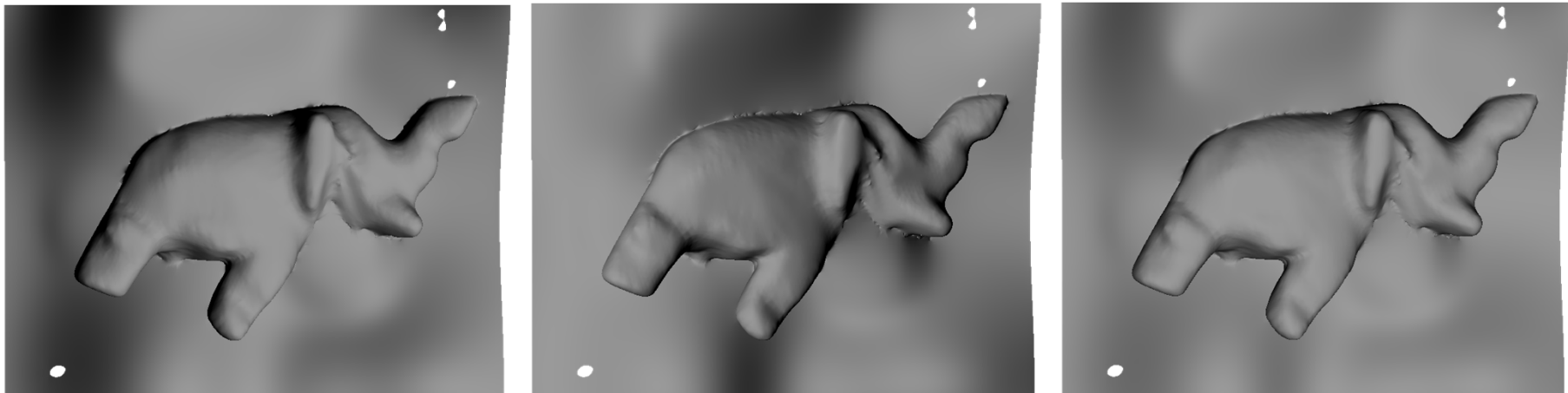
[Frankot, Chellappa

A method for enforcing integrability in SFS Alg.

PAMI 1998]

Example integrability

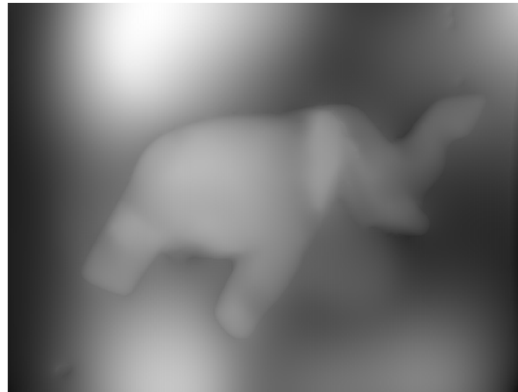
[Neil Birkbeck]



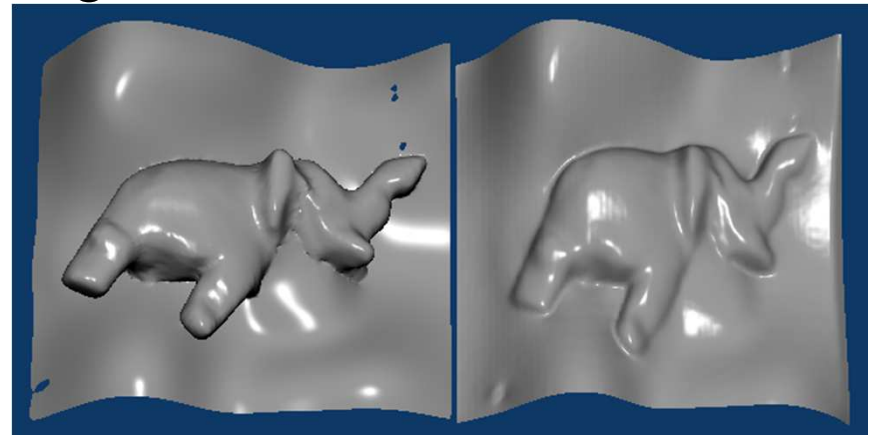
images with different light



normals



Integrated depth



original
surface

reconstructed

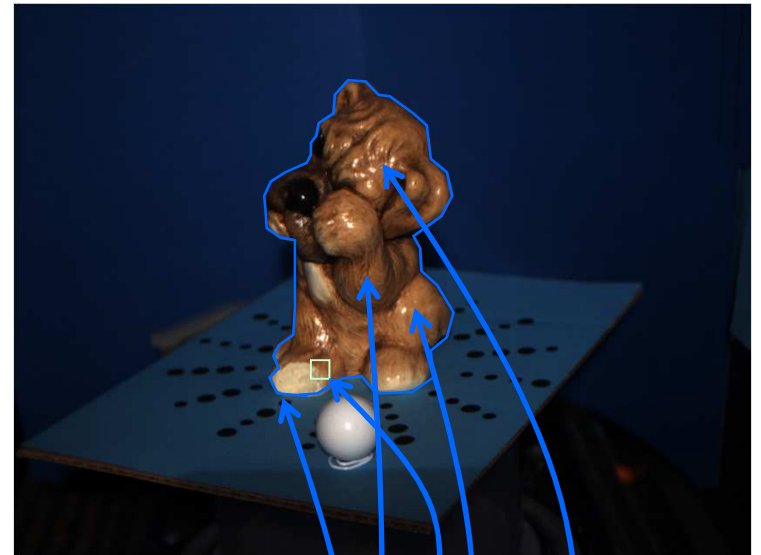
Image cues

Shading, Stereo, Specularities

Readings: See links on web page

Books: Szeliski 2.2, Ch 12

Forsythe Ch 4,5 (Lab related) .pdf
on web site)



Color (texture)
Shading
Shadows
Specular highlights
Silhouette

Upcoming

- Lab 4 due Mar31
- Exam 2: In-class Apr 2, same format as E1
 - Calculator and 4 sheets of your notes.
- Project presentations: Inclass Apr 9, 11
 - Present the motivation, related literature and libraries
 - Present your progress to-date
 - Prepare 5-10min presentation/person.
- Project report: Hand in at end of classes. (With an earlier hand-in I may have time to comment and you can polish it for a final hand-in)
- Project demos
 - Submit visual demos (videos and easily runnable code)
 - If you like can schedule an in-person demo.

All images

- Unknown lights and normals : It is possible to reconstruct the surface and light positions ?
- What is the set of images of an object under all possible light conditions ?



[Debevec et al]

Space of all images

Problem:

- Lambertian object
- Single view, orthographic camera
- Different illumination conditions (distant illumination)



1. 3D subspace:

[Moses 93][Nayar,Murase 96][Shashua 97] + convex obj (no shadows)

2. Illumination cone:

[Belhumeur and Kriegman CVPR 1996]

3. Spherical harmonic representation:

[Ramamoorthi and Hanharan Siggraph 01]

[Barsi and Jacobs PAMI 2003]

3D subspace

Convex cone

Linear
combination of
harmonic imag.
(practical 9D basis)

3D Illumination subspace

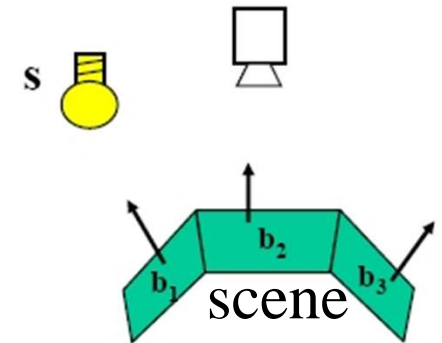
Lambertian reflection : $I = \rho \mathbf{n} \bullet \mathbf{l} = \mathbf{b} \bullet \mathbf{l}$

(one image point \mathbf{x})

Whole image :

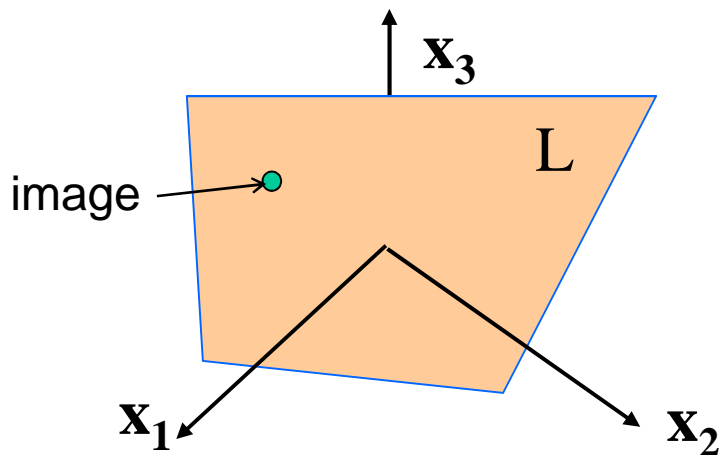
(image as vector \mathbf{I})

$$I(:) = \mathbf{x} = \mathbf{B}\mathbf{l} \quad \mathbf{B} = \begin{bmatrix} \mathbf{b}_1^T \\ \vdots \\ \mathbf{b}_n^T \end{bmatrix} \quad n \times 3$$



The set of images of a Lambertian scene surface with no shadowing is a subset of a 3D subspace. [Moses 93][Nayar,Murase 96][Shashua 97]

$$L = \{\mathbf{x} \mid \mathbf{x} = \mathbf{B}\mathbf{l}, \forall \mathbf{l} \in \mathbb{R}^3\}$$



$$\begin{array}{ccc} \text{All images} & & \text{basis} \quad \text{All lights} \\ \left[\begin{array}{c} \text{orange bars} \\ \mathbf{x}_1 \quad \mathbf{x}_2 \quad \mathbf{x}_3 \quad \mathbf{x}_4 \end{array} \right] & = & \left[\begin{array}{c} \text{green bars} \\ \mathbf{B} \\ \rho \mathbf{n}^T \end{array} \right] \left[\begin{array}{c} \text{yellow bars} \\ \mathbf{l}_1 \quad \mathbf{l}_2 \quad \mathbf{l}_3 \quad \mathbf{l}_4 \end{array} \right] \\ n \times m & & n \times 3 \quad 3 \times m \end{array}$$

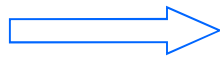
Reconstructing the basis

$$L = \{\mathbf{x} \mid \mathbf{x} = \mathbf{B}\mathbf{l}, \forall \mathbf{l} \in \mathbb{R}^3\}$$

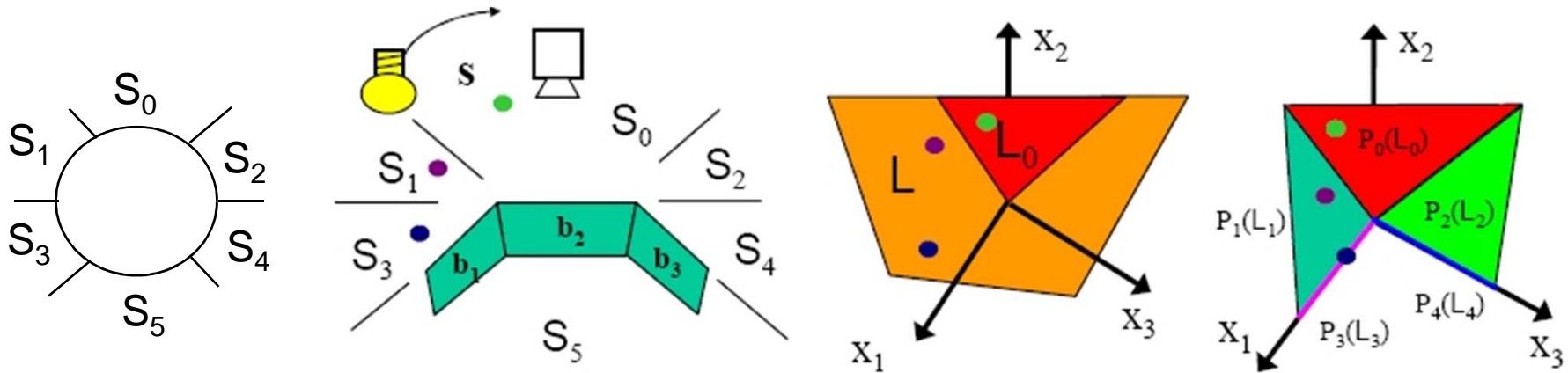
- Any three images without shadows span L.
- L – represented by an orthogonal basis B.
- How to extract B from images ?



PCA



Shadows



No shadows
Shadows

$$L = \{\mathbf{x} \mid \mathbf{x} = \mathbf{B}\mathbf{l}, \forall \mathbf{l} \in \mathbb{R}^3\}$$

$$\mathbf{x} = \max(\mathbf{B}\mathbf{l}, 0)$$

Ex: images with all pixels illuminated

$$L_0 = L \cap \{\mathbf{x} \mid \mathbf{x} \in \mathbb{R}^n, I_j \geq 0, \forall j\}$$

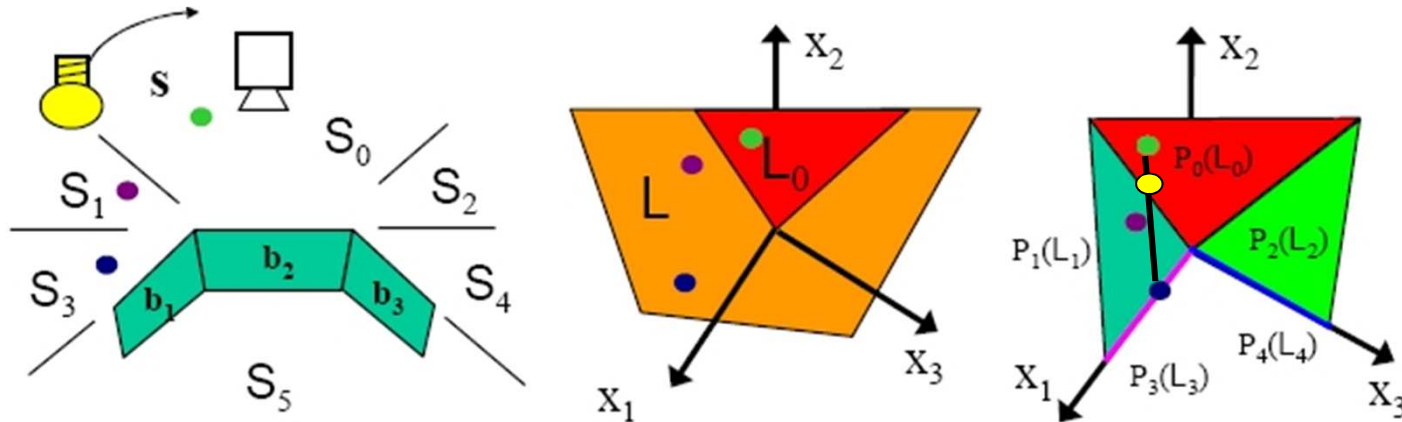
Single light source

- L_i intersection of L with an orthant i of \mathbb{R}^n
corresponding cell of light source directions S_i for which the same pixels are in shadow and the same pixels are illuminated.
- $P(L_i)$ projection of L_i that sets all negative components of L_i to 0 (convex cone)

The set of images of an object produces by a single light source is :

$$U = \{\mathbf{x} \mid \mathbf{x} = \max(\mathbf{B}\mathbf{l}, 0), \forall \mathbf{l} \in \mathbb{R}^3\} = \bigcup_i P_i(L_i)$$

Shadows and multiple lights



Shadows, multiple lights $\mathbf{x} = \sum_i \max(\mathbf{B}\mathbf{l}_i, 0)$

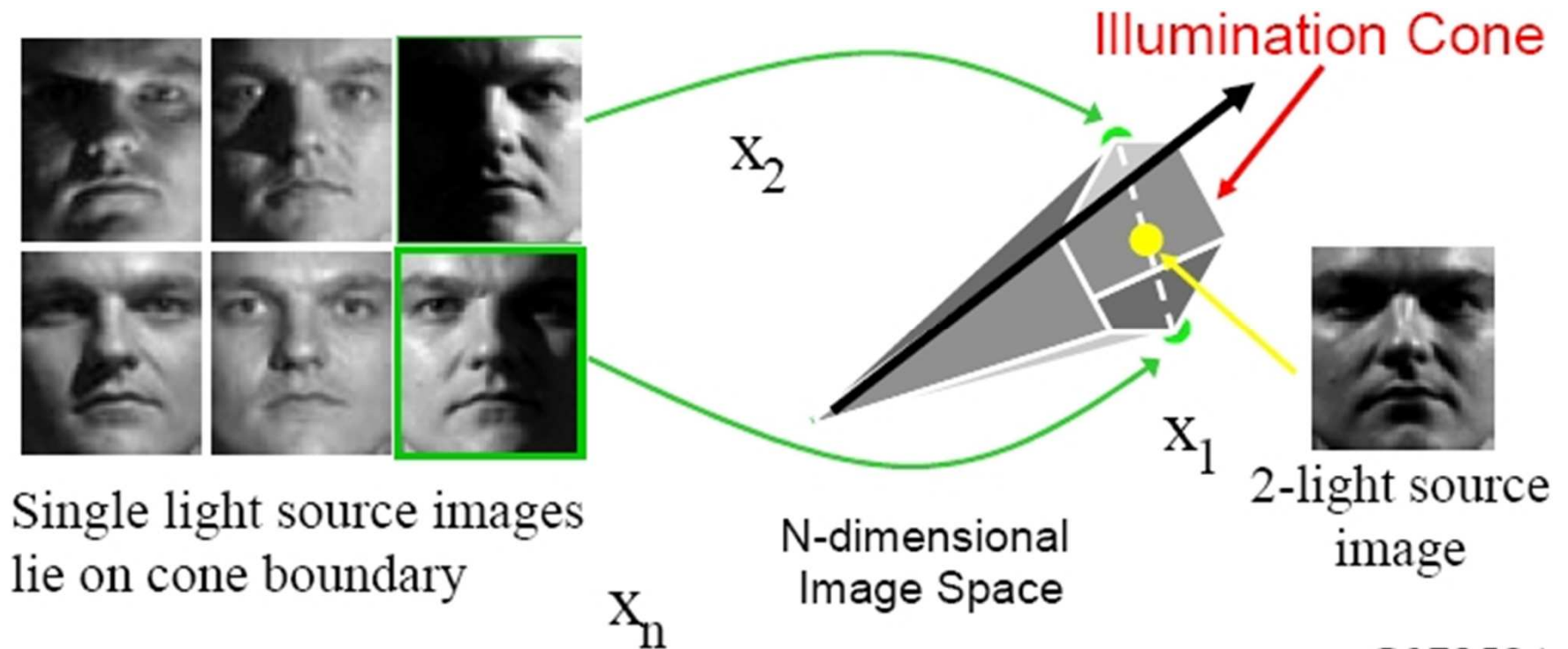
The image illuminated with two light sources l_1, l_2 , lies on the line between the images of x_1 and x_2 .

The set of images of an object produces by an arbitrary number of lights is the convex hull of $U =$ illumination cone C .

Illumination cone

The set of images of a any Lambertian object under all light conditions is a convex cone in the image space.

[Belhumeur, Kriegman: What is the set of images of an object under all possible light conditions ?, IJCV 98]

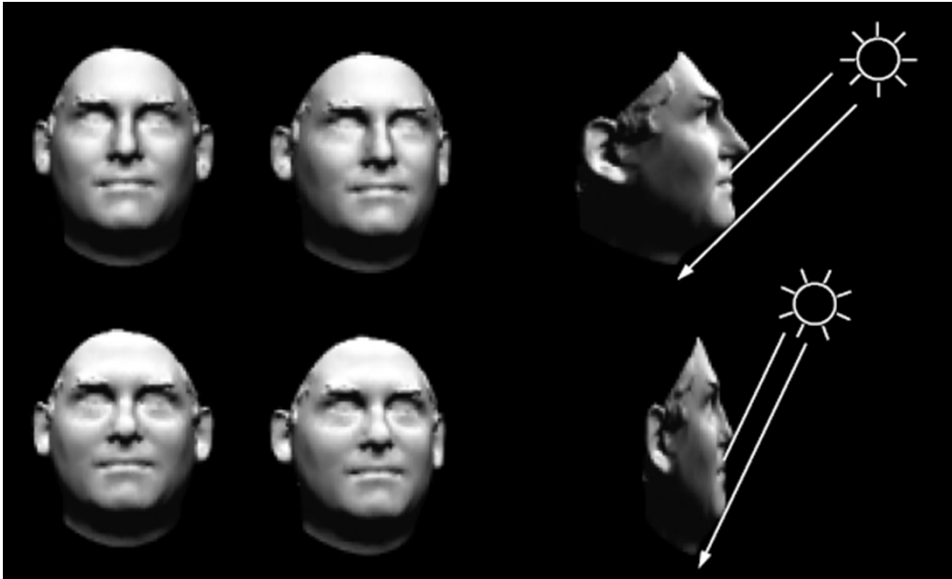


Do ambiguities exist ?

Can two different objects produce the same illumination cone ?

YES

“Bas-relief” ambiguity

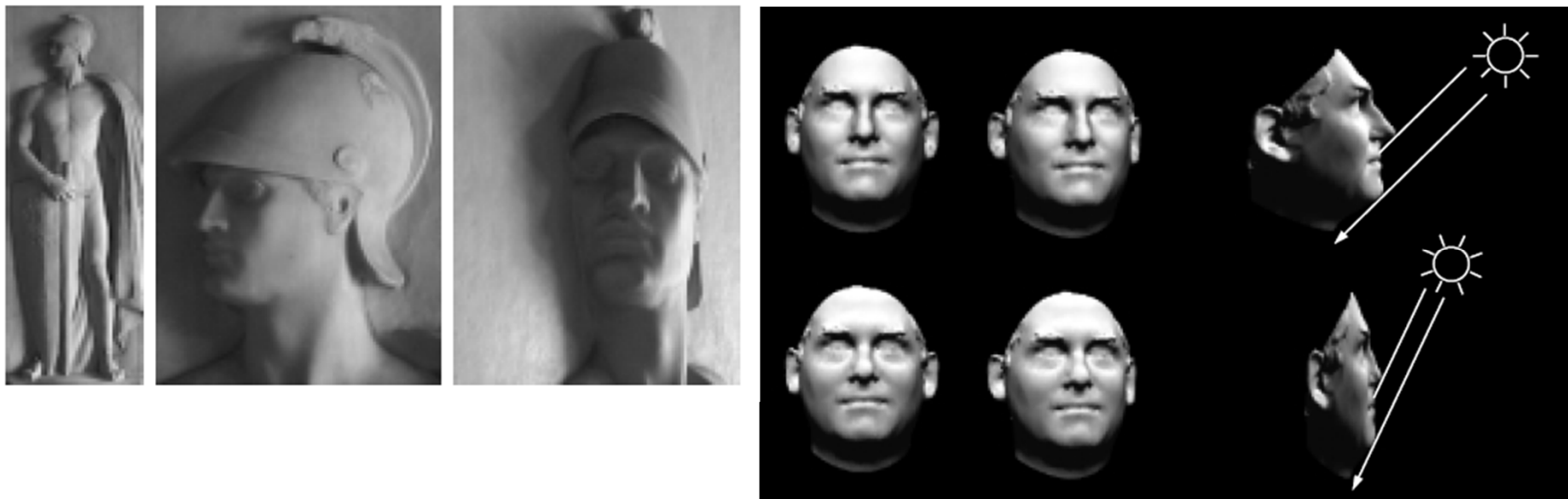


Convex object

- B span L
 - Any $A \in GL(3)$, $B^* = BA$ span L
 - $I = B^* S^* = (BA)(A^{-1}S) = BS$
- Same image B lighted with S
and B^* lighted with S^*

When doing PCA the resulting basis is generally not normal*albedo

GBR transformation



[Belhumeur et al: The bas-relief ambiguity IJCV 99]

Surface integrability :

Real B , transformed $B^* = BA$ is integrable only for General Bas Relief transformation.

$$A = G^T = \begin{bmatrix} \lambda & 0 & -\mu \\ 0 & \lambda & -v \\ 0 & 0 & 1 \end{bmatrix}$$

$$\bar{f}(x, y) = \lambda f(x, y) + \mu x + v y$$

Uncalibrated photometric stereo

- Without knowing the light source positions, we can recover shape only up to a GBR ambiguity.

1. From n input images compute B^* (SVD)
2. Find A such that $B^* A$ close to integrable
3. Integrate normals to find depth.

Comments

- GBR preserves shadows [[Kriegman, Belhumeur 2001](#)]
- If albedo is known (or constant) the ambiguity G reduces to a binary subgroup [[Belhumeur et al 99](#)]
- Interreflections : resolve ambiguity [[Kriegman CVPR05](#)]

Spherical harmonic representation

Theory : infinite no of light directions
space of images infinite dimensional

[Illumination cone, Belhumeur and Kriegman 96]

Practice : (empirical) few bases are enough

[Hallinan 94, Epstein 95]


$$\text{Target Image} = .2 \times \text{Basis Image 1} + .3 \times \text{Basis Image 2} + \dots$$



Simplification : Convex objects (no shadows, interreflections)

[Ramamoorthi and Hanharan: Analytic PCA construction for Theoretical analysis of Lighting variability in images of a Lambertian object: SIGGRAPH01]

[Barsi and Jacobs: Lambertian reflectance and linear subspaces: PAMI 2003]

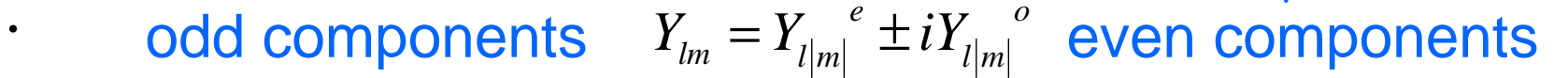
Basis approximation



Spherical harmonics basis

- Sphere analog to the Fourier basis on the line or square
- Angular portion of the solution to Laplace equation in spherical coordinates $\nabla^2 \psi = 0$
- Orthonormal basis for the set of all functions on the surface of the sphere

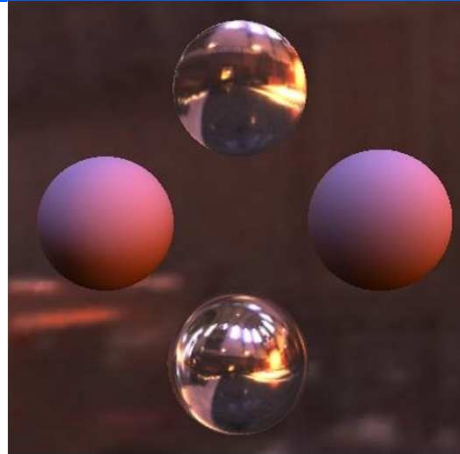
$$Y_{lm}(\theta, \phi) = \underbrace{\sqrt{\frac{(2l+1)}{4\pi} \frac{(l-|m|)!}{(l+|m|)!}}}_{\text{Normalization factor}} \underbrace{P_{l|m|}(\cos \theta) e^{im\phi}}_{\substack{\text{Legendre} \\ \text{functions} \quad \text{Fourier} \\ \text{basis}}}$$



Example of approximation



Exact image



9 terms approximation

Efficient rendering

- known shape
- complex illumination (compressed)



[Ramamoorthi and Hanharan: An efficient representation for irradiance environmental map Siggraph 01]

Not good for high frequency (sharp) effects ! (specularities)

Relation between SH and PCA

[Ramamoorthi PAMI 2002]

Prediction: 3 basis 91% variance
5 basis 97%

Empirical: 3 basis 90% variance
5 basis 94%



42%



33%



16%

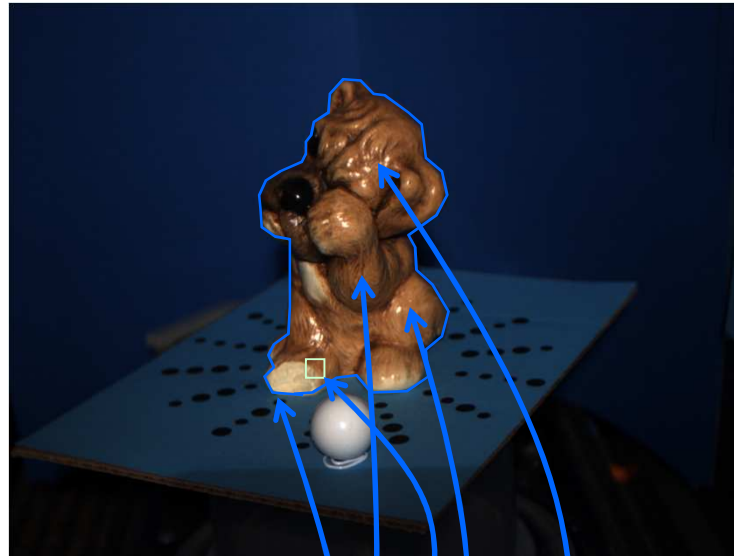


4%



2%

Summary: Image cues



Color (texture)
Shading
Shadows
Specular highlights
Silhouette

Properties of SH

Function decomposition

f piecewise continuous function on the surface of the sphere

$$f(u) = \sum_{l=0}^{\infty} \sum_{m=-l}^l f_{lm} Y_{lm}(u)$$

where

$$f_{lm} = \int_{S^2} f(u) Y_{lm}^*(u) du$$

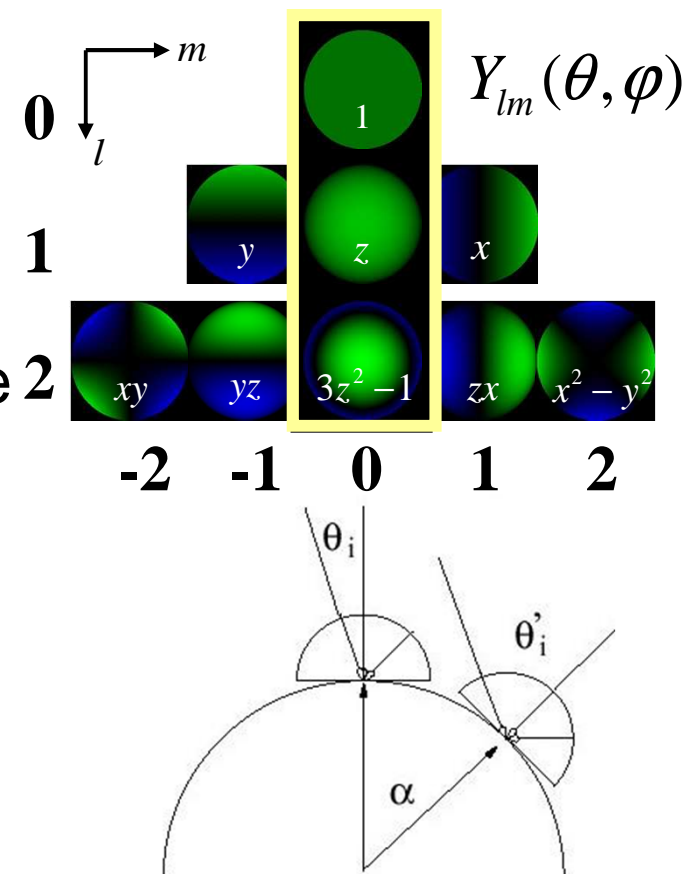
Rotational convolution on the sphere

Funk-Hecke theorem:

k circularly symmetric bounded integrable function on $[-1,1]$

$$k(u) = \sum_{l=0}^{\infty} k_l Y_{l0}$$

$$k * Y_{lm} = \alpha_l Y_{lm} \quad \alpha_l = \sqrt{\frac{4\pi}{2l+1}} k_l$$



Reflectance as convolution

Lambertian reflectance

One light $R(u') = l(u) \rho \max(0, u \bullet u')$

Lambertian kernel $k(u \bullet u') = \max(0, u \bullet u')$

Integrated light $R(u') = \int_{S^2} k(u \bullet u') l(u) du$

SH representation

light

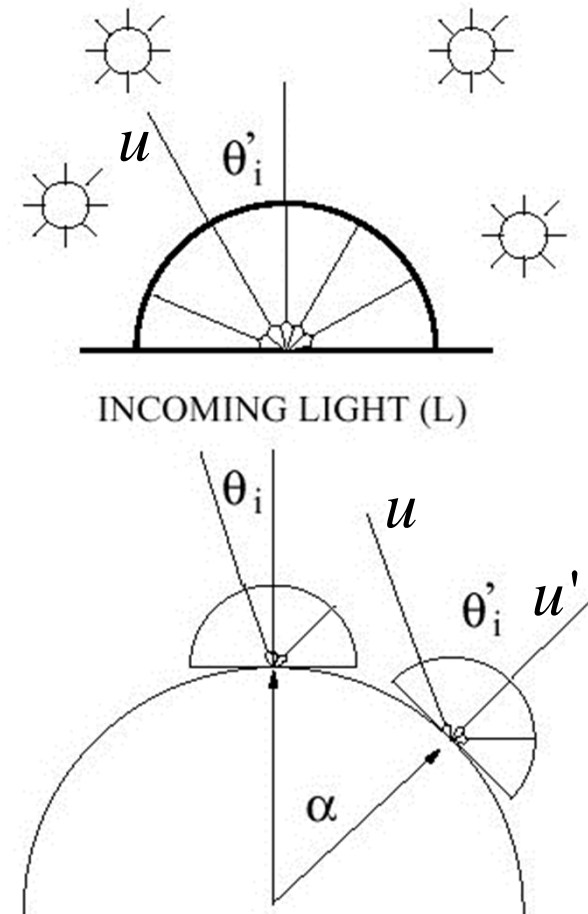
$$l(u) = \sum_{l=0}^{\infty} \sum_{m=-l}^l l_{lm} Y_{lm}(u)$$

Lambertian kernel

$$k = \sum_{l=0}^{\infty} k_l Y_{l0}$$

Lambertian reflectance (convolution theorem)

$$R = k * l = \sum_{l=0}^{\infty} \sum_{m=-l}^l \left(\sqrt{\frac{4\pi}{2l+1}} k_l l_{lm} \right) Y_{lm} = \sum_{l=0}^{\infty} \sum_{m=-l}^l r_{lm} Y_{lm}$$



Convolution kernel

Lambertian kernel

$$k(u \bullet u') = \max(0, u \bullet u')$$

$$k = \sum_{l=0}^{\infty} k_l Y_{l0}$$

$$k_l = \begin{cases} \frac{\sqrt{\pi}}{2} & n=0 \\ \frac{\sqrt{\pi}}{3} & n=1 \\ (-1)^{l/2+1} \frac{\sqrt{(2l+1)\pi}}{2^l(l-1)(l+2)} \binom{l}{l/2} & n \geq 2, \text{even} \\ 0 & n \geq 2, \text{odd} \end{cases}$$

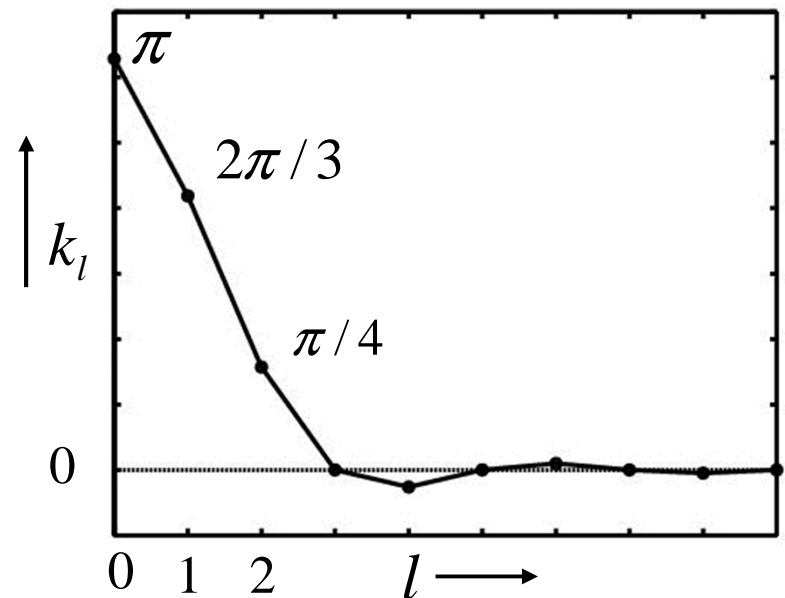
Asymptotic behavior of k_l for large l

$$k_l \approx l^{-2} \quad r_{lm} \approx l^{-5/2}$$

- Second order approximation accounts for 99% variability
- k like a low-pass filter

[Basri & Jacobs 01]

[Ramamoorthi & Hanrahan 01]



From reflectance to images

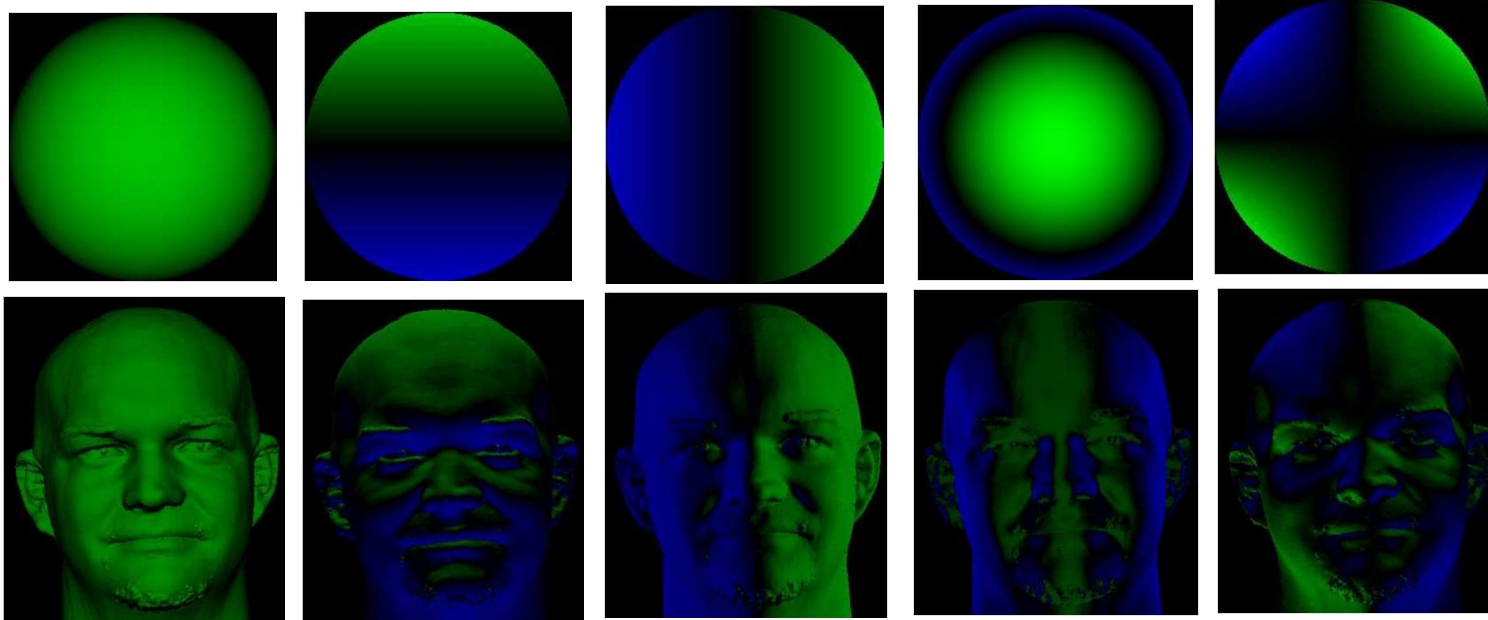
Unit sphere \Rightarrow general shape
Rearrange normals on the sphere

Reflectance on a sphere

$$R = k * l = \sum_{l=0}^{\infty} \sum_{m=-l}^l r_{lm} Y_{lm}$$

Image point with normal n_i

$$I_i = \sum_{l=0}^{\infty} \sum_{m=-l}^l \rho_i r_{lm} Y_{lm}(n_i)$$



Shape from Shading

Given: **one** image of an object illuminated with a distant light source

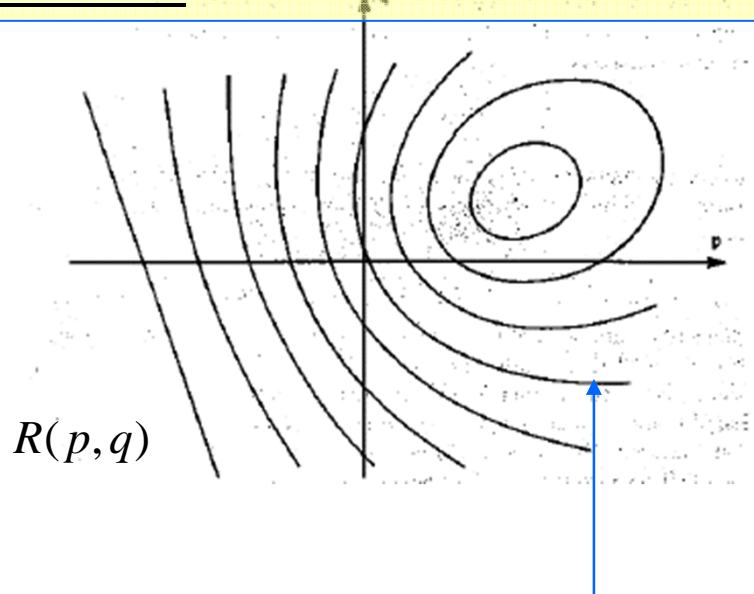
Assume: Lambertian object, with known, or constant albedo (usually assumes 1)

orthographic camera

known light direction

ignore shadows, interreflections

Recover: normals



Radiance of one pixel constrains
the normal to a curve

ILL-POSED

Surface $s(x, y) = (x, y, f(x, y))$

Gradient space $p = \frac{\partial f}{\partial x} \quad q = \frac{\partial f}{\partial y}$

Normal $\mathbf{n} = (p, q, -1)$

$$\hat{\mathbf{n}} = \frac{1}{\sqrt{p^2 + q^2 + 1}}(p, q, -1)$$

Lambertian reflectance: depends
only on $\mathbf{n}(p, q)$:

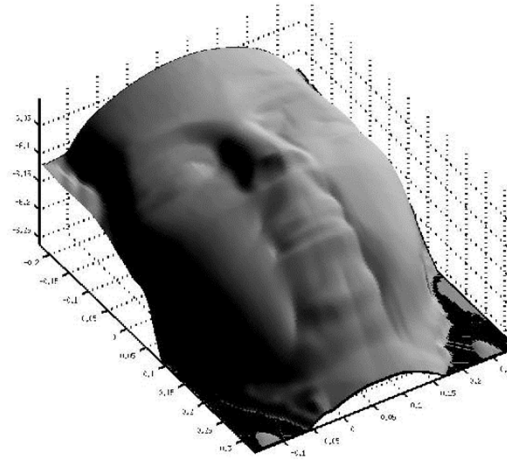
$$E(\mathbf{x}) = \cos(\mathbf{n}(\mathbf{x}), \mathbf{l}) = \frac{\mathbf{n}(\mathbf{x}) \cdot \mathbf{l}}{\|\mathbf{n}(\mathbf{x})\|}$$

Variational SFS



Image info

shading



Recovers

Integrated normals

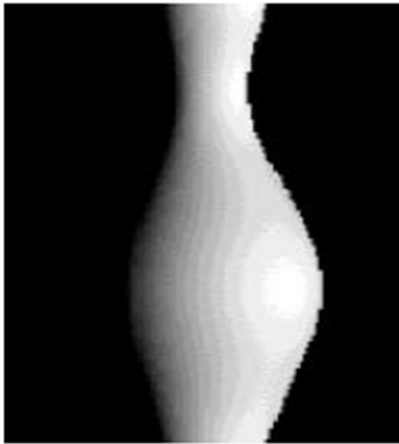
- Defined by [Horn](#) and others in the 70's.
- Variational formulation

$$\iint_{\text{object}} (I(x, y) - E(p, q))^2 dx dy = \iint_{\text{object}} \left(I(x, y) - \frac{[p, q, -1] \cdot \mathbf{l}}{\sqrt{p^2 + q^2 + 1}} \right)^2 dx dy + \alpha \iint_{\text{object}} \left(\frac{\partial p}{\partial y} - \frac{\partial q}{\partial x} \right)^2 dx dy$$

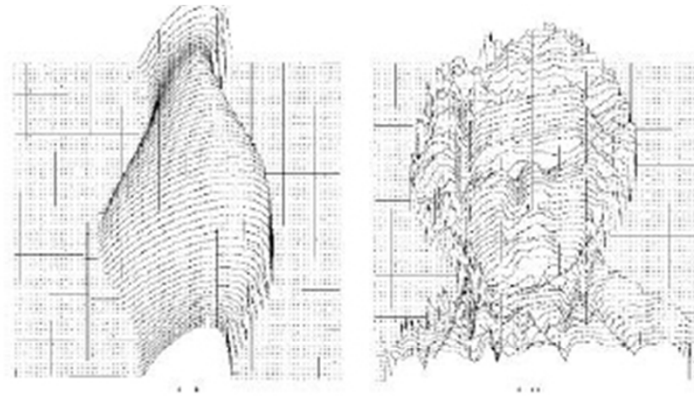
regularization

- Showed to be ill-posed [[Brooks 92](#)] (ex. *Ambiguity convex/concave*)
- Classical solution – add regularization, integrability constraints
- Most published algorithms are non-convergent [[Duron and Maitre 96](#)]

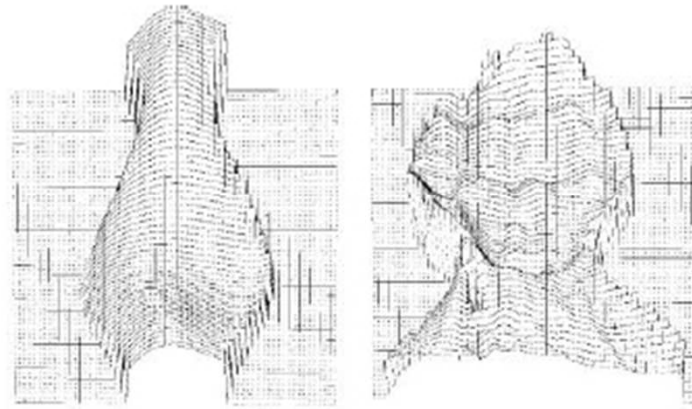
Examples of results



Synthetic images



Tsai and Shah's method 1994



Pentland's method 1994

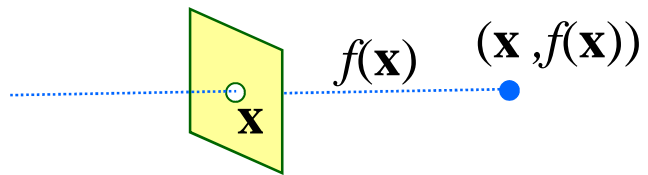
Well posed SFS

[Prados ICCV03, ECCV04] reformulated SFS as a well-posed problem

$$E(\mathbf{x}) = \cos(\mathbf{n}(\mathbf{x}), \mathbf{L}) = \frac{\mathbf{n}(\mathbf{x}) \cdot \mathbf{L}}{|\mathbf{n}(\mathbf{x})|}$$

Lambertian reflectance

Orthographic camera

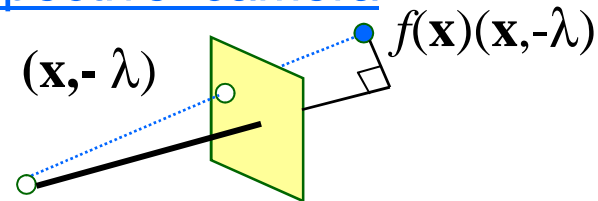


$$s = \{(\mathbf{x}, f(\mathbf{x})) \mid \mathbf{x} = (u, v) \in \Omega\}$$

$$\mathbf{n}(s(\mathbf{x})) = (\nabla f(\mathbf{x}), -1)$$

$$I(\mathbf{x}) = \frac{-\nabla f(\mathbf{x}) \cdot \mathbf{l} + c}{\sqrt{1 + |\nabla f(\mathbf{x})|^2}} \quad \mathbf{L} = (\mathbf{l}, c)$$

Perspective camera



$$s = \{f(\mathbf{x})(\mathbf{x}, -\lambda) \mid \mathbf{x} = (u, v) \in \Omega\}$$

$$\mathbf{n}(s(\mathbf{x})) = (\lambda \nabla f(\mathbf{x}), f(\mathbf{x}) + \mathbf{x} \cdot \nabla f(\mathbf{x}))$$

$$I(\mathbf{x}) = \frac{\lambda \mathbf{l} \cdot \nabla f(\mathbf{x}) + c(f(\mathbf{x}) + \mathbf{x} \cdot \nabla f(\mathbf{x}))}{\sqrt{\lambda^2 |\nabla f(\mathbf{x})|^2 + (f(\mathbf{x}) + \mathbf{x} \cdot \nabla f(\mathbf{x}))^2}}$$

Hamilton-Jacobi equations - no smooth solutions;

$$H(x, \nabla u) = 0 \quad - \text{require boundary conditions}$$

Well-posed SFS (2)

Hamilton-Jacobi equations - no smooth solutions;
- require boundary conditions

Solution

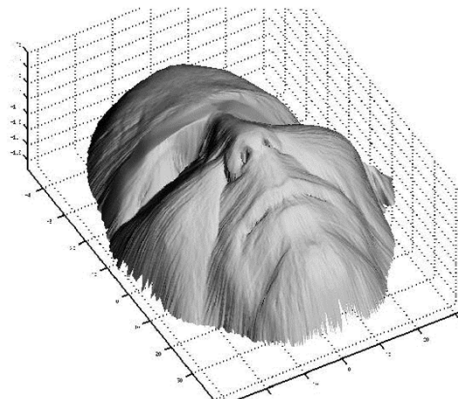
1. **Impose smooth solutions** – not practical because of image noise
2. **Compute viscosity solutions** [Lions et al.93] (smooth almost everywhere)
still require boundary conditions

E. Prados :general framework – characterization viscosity solutions.

(based on Dirichlet boundary condition)

efficient numerical schemes for orthogonal and perspective camera

showed that SFS is a well-posed for a finite light source



[Prados ECCV04]

Shading: Summary

Space of all images :

Lambertian object
Distant illumination
One view (orthographic)

+ Convex objects

3D subspace

1. 3D subspace

2. Illumination cone:

Convex cone

2. Spherical harmonic representation:

Linear combination
of harmonic imag.
(practical 9D basis)

Reconstruction :

Single light source

One image
Unit albedo
Known light

Ill-posed
+ additional
constraints

1. Shape from shading

2. Photometric stereo

Multiple imag/1 view
Arbitrary albedo
Known light

3. Uncalibrated photometric stereo

+ Unknown light

GBR ambiguity
Family of solutions

Extension to multiple views

Problem: PS/SFS one view \Rightarrow incomplete object

Solution: extension to multiple views – rotating obj., light var.

Problem: we don't know the pixel correspondence anymore

Solution: iterative estimation: normals/light – shape

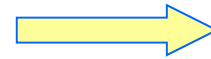
initial surface from SFM or visual hull



Input images



Initial surface



Refined surface

1. Kriegman et al ICCV05; Zhang, Seitz ... ICCV 03

SFM

2. Cipolla, Vogiatzis ICCV05, CVPR06

Visual hull

Multiview PS+ SFM points

[Kriegman et al ICCV05][Zhang, Seitz ... ICCV 03]

1. SFM from corresponding points:
camera & initial surface (Tomasi
Kanade)

2. Iterate:

- factorize intensity matrix : light, normals, GBR ambiguity
- Integrate normals
- Correct GBR using SFM points (constrain surface to go close to points)



images

Initial

surface

$$I = \mathbf{L}\mathbf{N}$$
$$\sum_{xy} \left(\frac{\partial f}{\partial x} + \frac{n_x}{n_z} \right)^2 + \left(\frac{\partial f}{\partial y} + \frac{n_y}{n_z} \right)^2$$



Integrated
surface



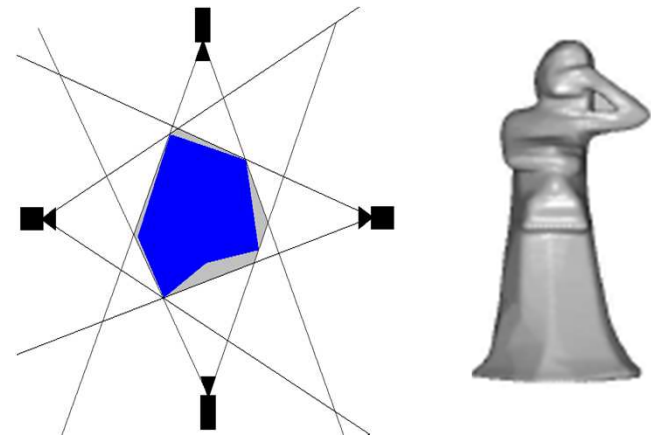
Rendered
Final surface

Multiview PS + frontier points

[Cipolla, Vogiatzis ICCV05, CVPR06]

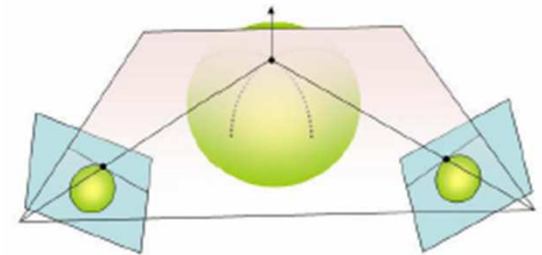
1. initial surface SFS

visual hull – convex envelope of the object



2. initial light positions from frontier points

plane passing through the point and the camera center is tangent to the object > known normals



3. Alternate photom normals / surface (mesh)

v *photom normals*

n *surface normals – using the mesh*

mesh –occlusions, correspondence in I

$$\sum_f \sum_i (l_i \bullet v_f - i_{fi})^2$$

$$\sum_f |n_f - v_f|^2$$

Multiview PS + frontier points



(a) Input images.



(b) Visual hull reconstruction.



(c) Our results.



(a) Input images.

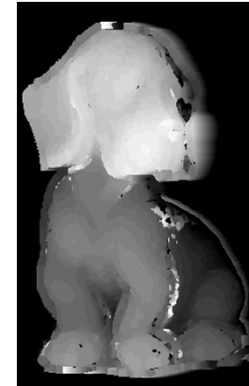
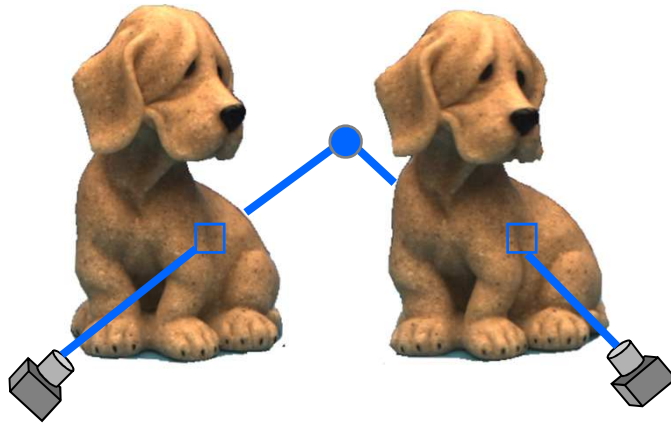


(b) Visual hull reconstruction.



(c) Our results.

Stereo



[Birkbeck]

[Assumptions two images

Lambertian reflectance

textured surfaces]

Image info

texture

Recover

per pixel depth

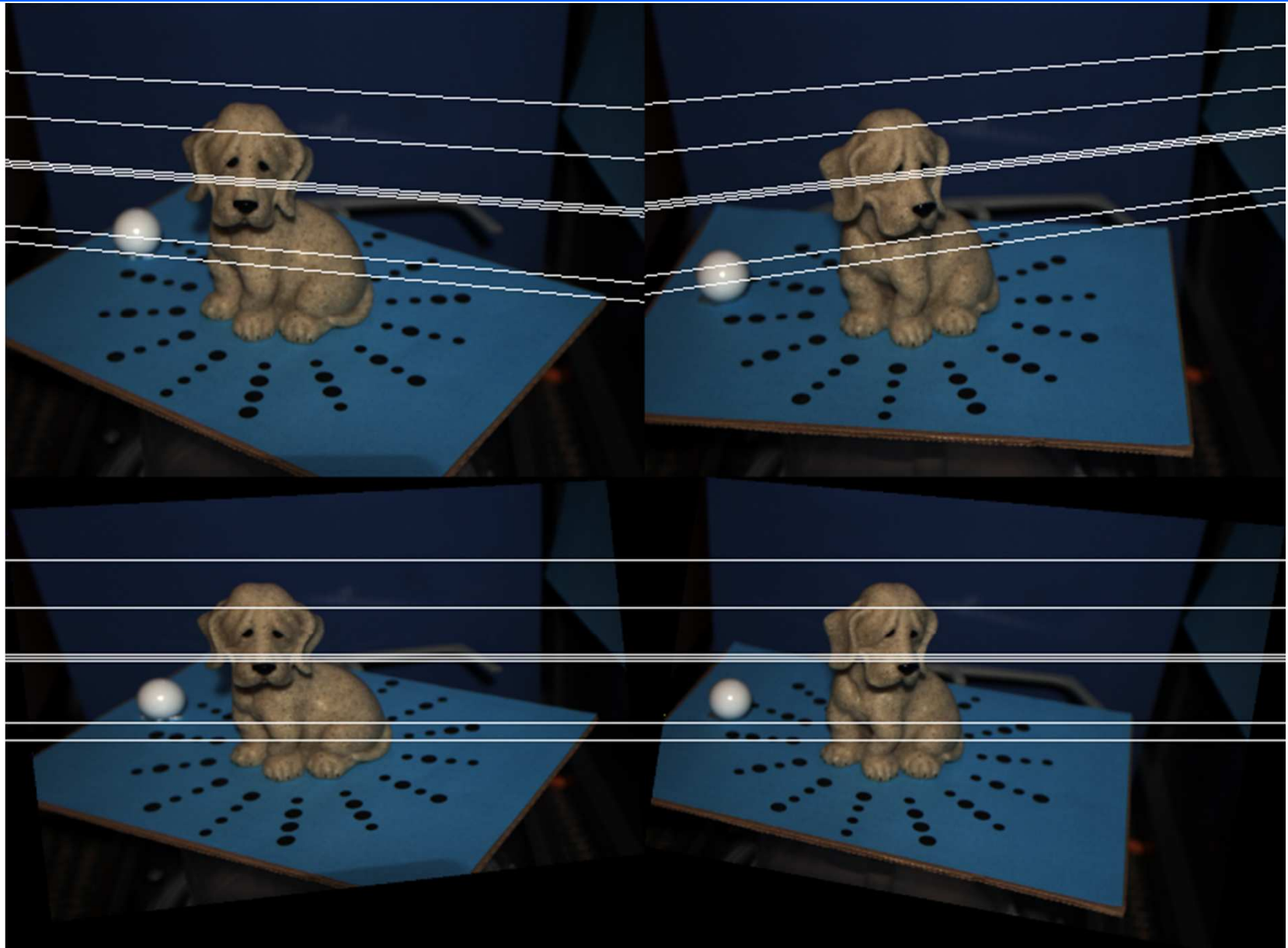
Approach

triangulation of corresponding points

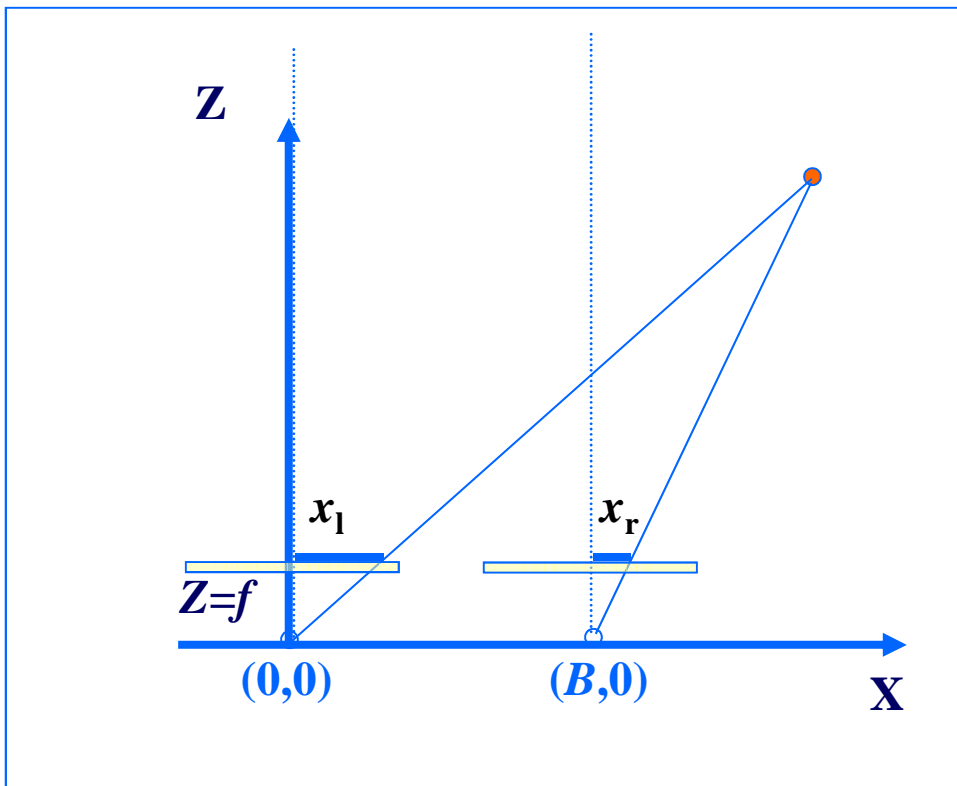
corresponding points

- recovered correlation of small patches around each point
- calibrated cameras – search along epipolar lines

Rectified images



Disparity



Disparity d

$$Z = \frac{f}{x_l} X = \frac{f}{x_r} (X - B)$$

$$Z = \frac{Bf}{\boxed{x_l - x_r}} = \frac{Bf}{\underset{d}{d}}$$

Correlation scores

Point:

$\mathbf{x} = (x, y, f(x, y))$ With respect to first image

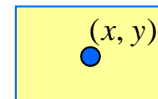
Calibrated cameras:

pixel in I_1 $m_1 = P_1(\mathbf{x}) = (x, y)$

pixel in I_2 $m_2 = P_2(\mathbf{x})$

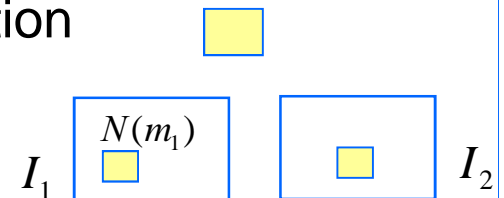
Small planar patch:

$N(x, y)$



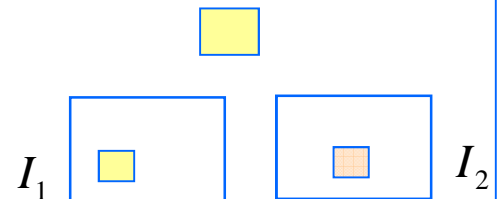
1. Plane parallel with image planes, no illumination variation

$$SAD_{12} = \int_{m \in N(m_1)} I_1(m_1 + m) - I_2(m_2 + m) dm$$



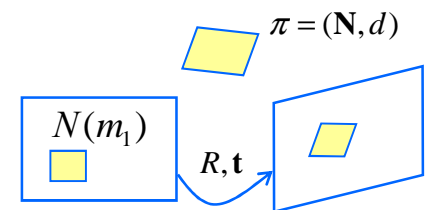
2. Compensate for illumination change

$$NCC_{12} = \int_{m \in N(m_1)} C_1(m) C_2(m) dm \quad C_i(m) = I_i(m_i + m) - \bar{I}_i(m_i) \quad \bar{I}_i \text{ mean}$$



3. Arbitrary plane

$$\int_{m \in N(m_1)} I_1(m_1 + m) - I_2(H(m_1 + m)) dm \quad H = R - \frac{\mathbf{t}\mathbf{N}^T}{d}$$



Specular surfaces

Reflectance equation

require: BRDF, light position

$$R_o = \rho(\theta_i, \phi_i, \theta_o, \phi_o) l(\theta_i, \phi_i) \cos(\theta_i)$$

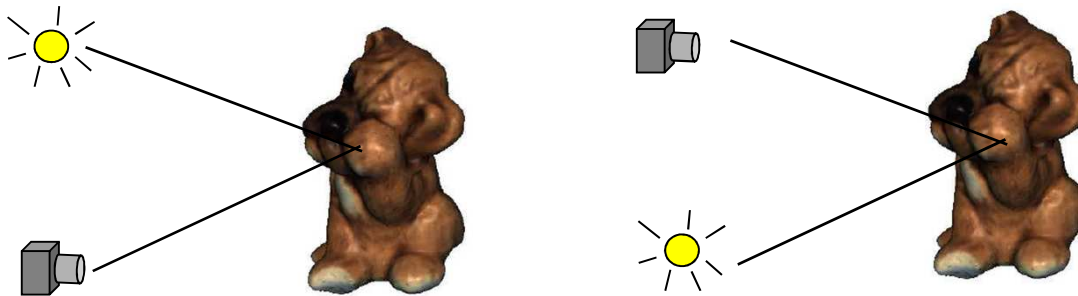
Image info

shading+specular highlights

Approaches

1. Filter specular highlights (*brightness, appear at sharp angles*)
2. Parametric reflectance
3. Non-parametric reflectance map (*discretization of BRDF*)
4. Account for general reflectance

Helmholz reciprocity [Magda et al ICCV 01, IJCV03]



Shape and Materials by Example

[Hertzmann, Seitz CVPR 2003 PAMI 2005]

Reconstructs objects with general BRDF with no illumination info.

Idea : A reference object from the same material but with known geometry (sphere) is inserted into the scene.



Reference images



Multiple materials



Results

Summary of image cues

	Reflectance	Light	+	-
stereo	textured Lambertian	Constant [SAD]	Rec. texture	Needs texture
		Varying [NCC]	Rec. depth discont. Complete obj	Occlusions
shading	uniform Lamb	Constant [SFS]		Uniform material Not robust Needs light pose
	unif/textured Lamb	Varying [PS]	Unif/varying albedo	Do not reconstr depth disc., one view