Projects

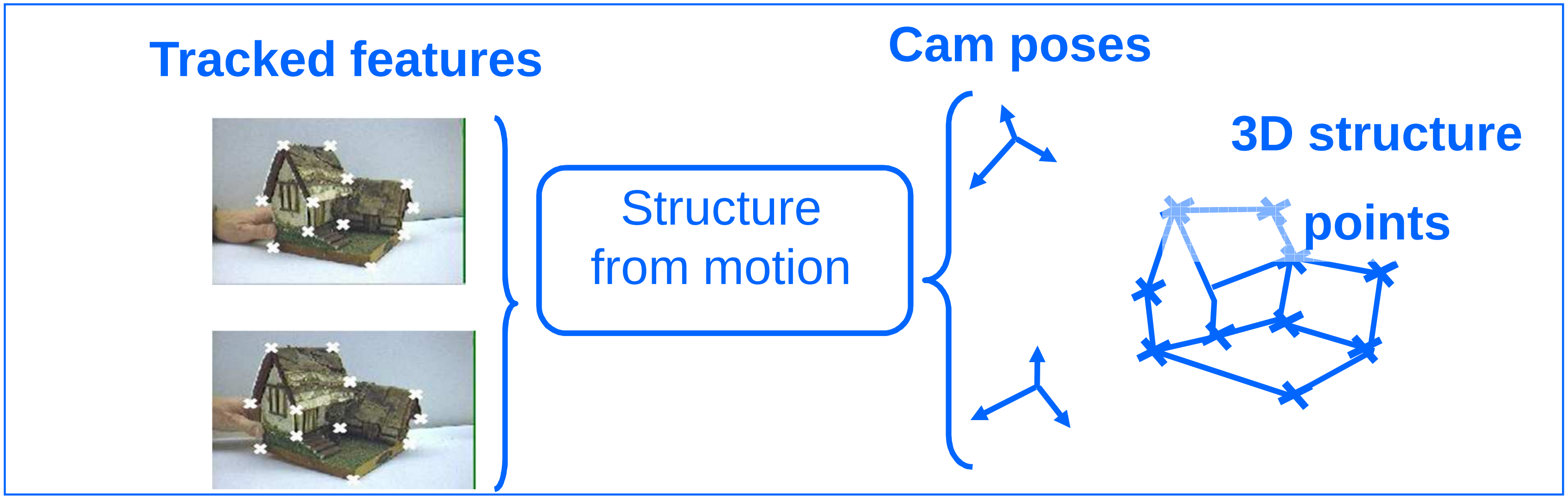
- Any Q/A or any help TA and I can provide?
- Feel free to engage in dialogue with TA/me on how to proceed
- Think about how to connect the project to course material.
- Interaction encouraged.
  - Attribute contributions to the people/sources

Light and Reflectance

Dana Cobzas Neil Birkbeck Martin Jagersand

most of course until now …

- SFM to reconstruct 3D points from 2D feature points (camera geometry, projective spaces …)
  - Feature correspondence : correlation, tracking assumes image constancy – constant illumination, no specularities, complex material
  - 10,100 or even 1000 3D points is not a complete scene or object model
- No notion of object surface
- No notion of surface properties (reflectance)



Tracked features

Structure from motion

Cam poses

3D structure

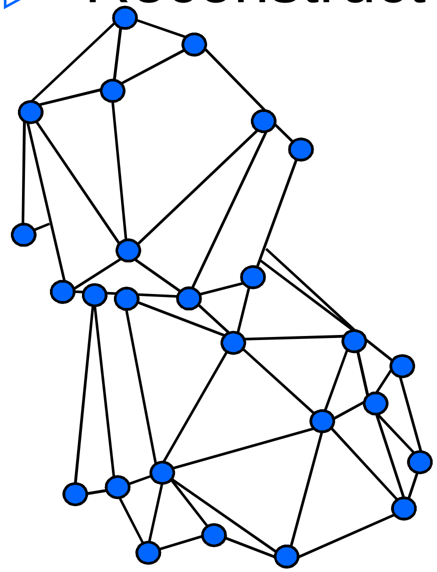
points

Now ...

- View surface as a whole – different surface representations
- Consider interaction of surface with light – explicitly model light, reflectance, material properties

⇨ Reconstruct whole objects = surface (detailed geometry)

⇨ Reconstruct material properties = reflectance



SFM

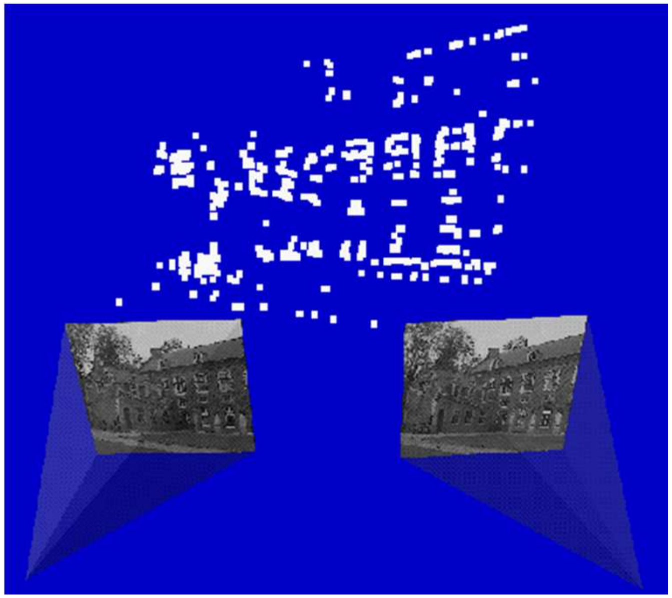
Surface estimation

Ex CapGui obj

Reflectance estimation

Stereo reconstruction

How to go from sparse SFM



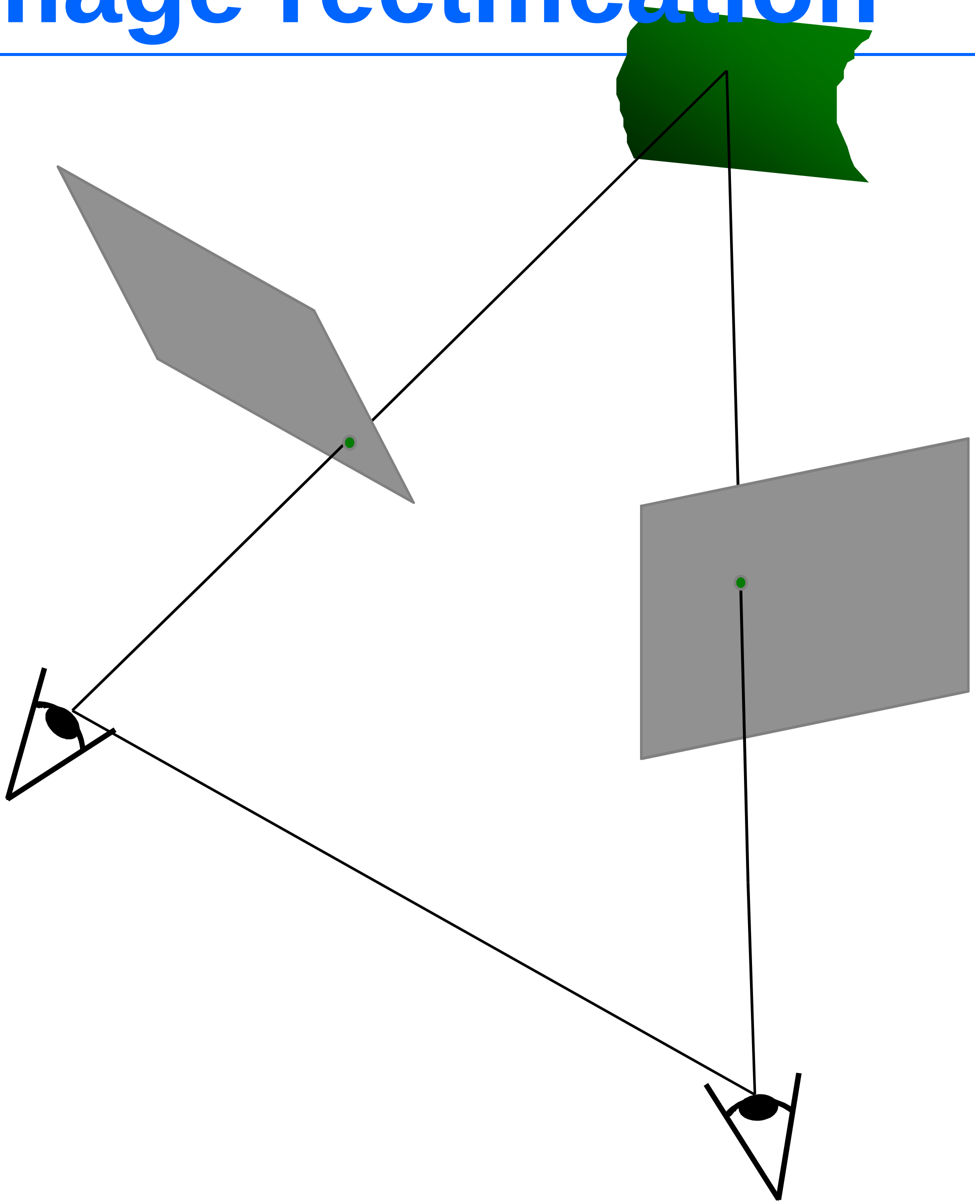
…to detailed, model?
Here in the form of
disparity/depth map

Rectified left image I(x,y)

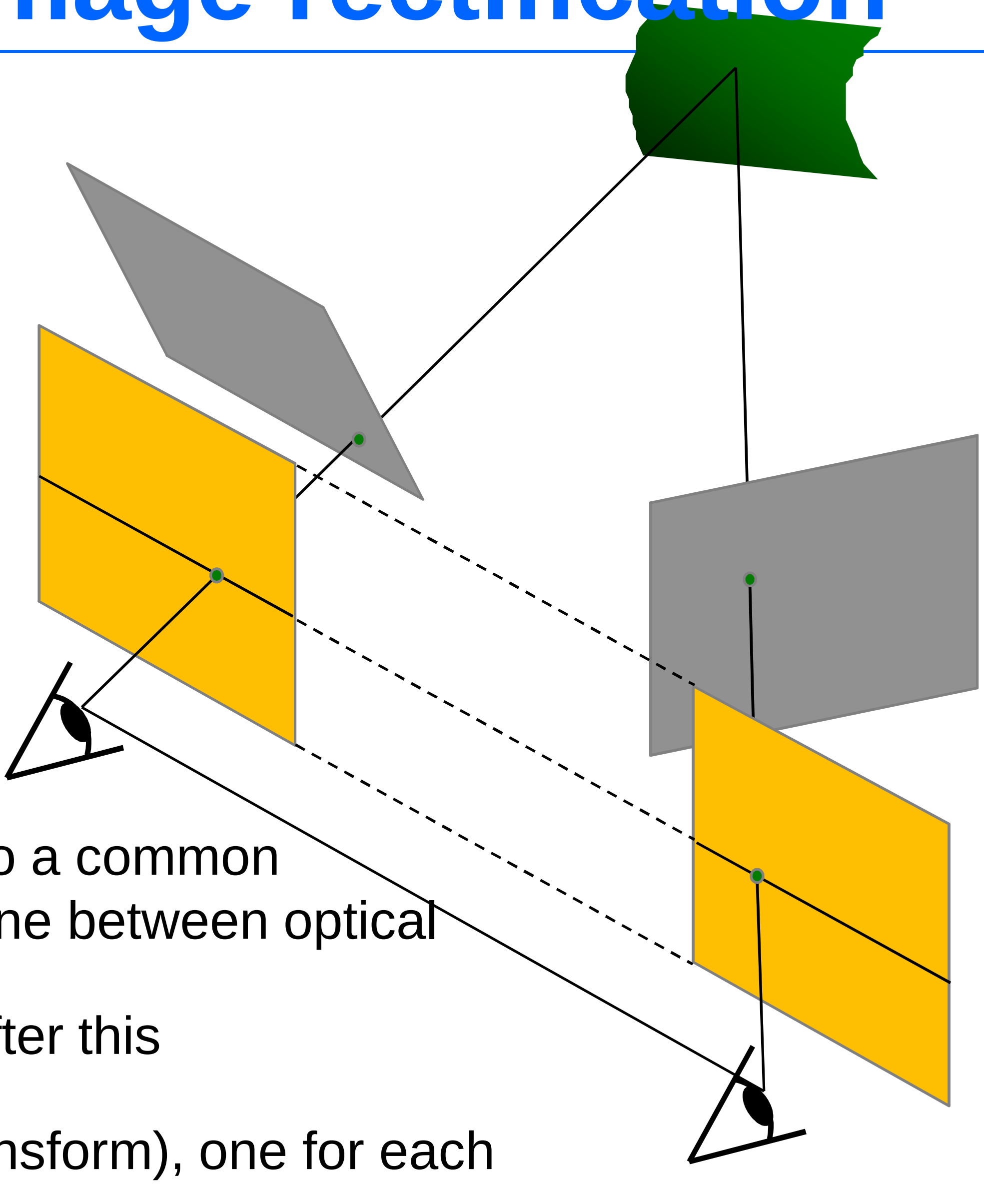
Dense Disparity map D(x,y)

Rectified right image I´(x´,y´)

Stereo image rectification



Stereo image rectification

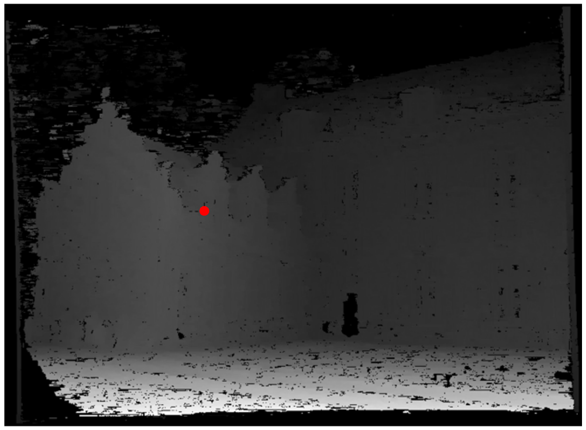


- reproject image planes onto a common
- plane parallel to the line between optical centers
- pixel motion is horizontal after this transformation
- two homographies (3x3 transform), one for each input image reprojection

Dense Stereo reconstruction

Rectified left
image I(x,y)

Dense Disparity map D(x,y)



Rectified right
image I´(x´,y´)

$$(x', y') = (x + D(x,y), y)$$

D is a “depth image)

(not full 3D model)

Brief outline

- Image formation - camera, light,reflectance
- Radiometry and reflectance equation
- BRDF
- Light models and inverse light
- Shading, Interreflections

Lec 1

- Image cues – shading
    - Photometric stereo
    - Shape from shading
- Image cues – stereo
- Image cues – general reflectance

Lec 2

- Multi-view methods
    - Volumetric – space carving
    - Graph cuts
    - Variational stereo
    - Level sets
    - Mesh

Lec 3,4

Lecture 1

Radiometry
Light and Reflectance

Image formation

Image

Shading

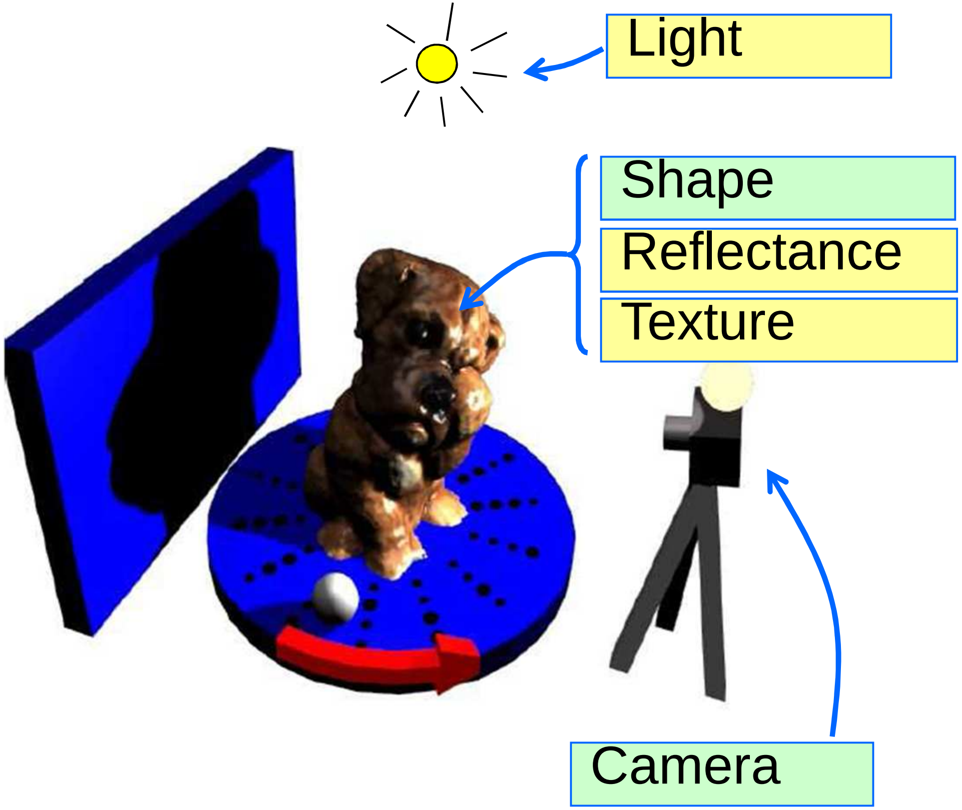
Shadows

Specular highlights

[Intereflections]

[Transparency]

Image formation



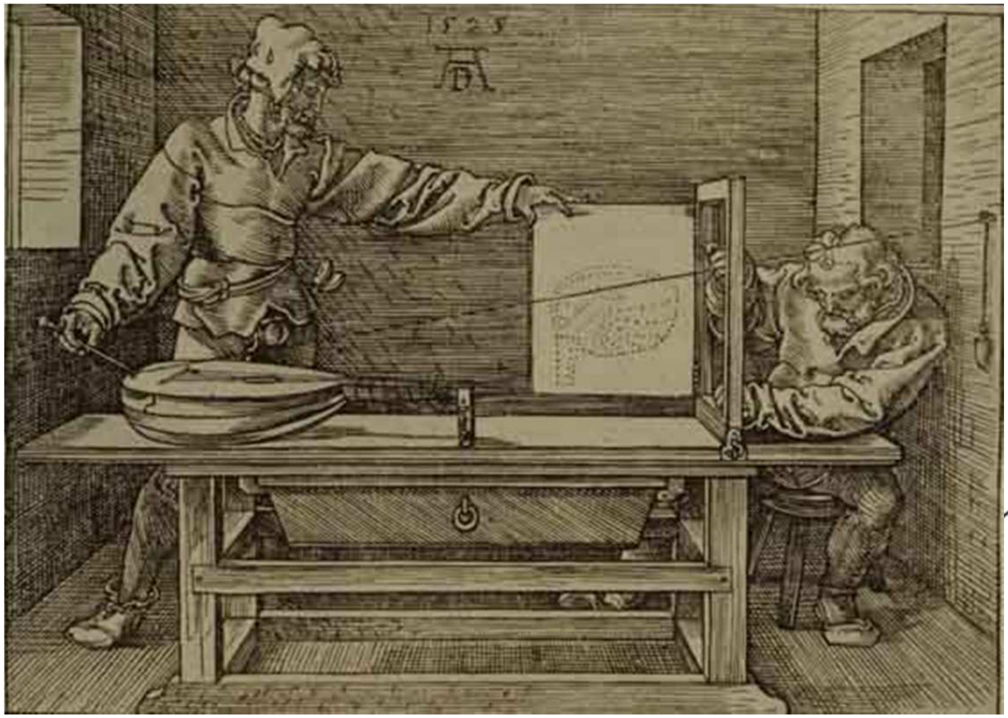
Images 2D + [3D shape] ⇨ **Light** [+ Reflectance+Texture]

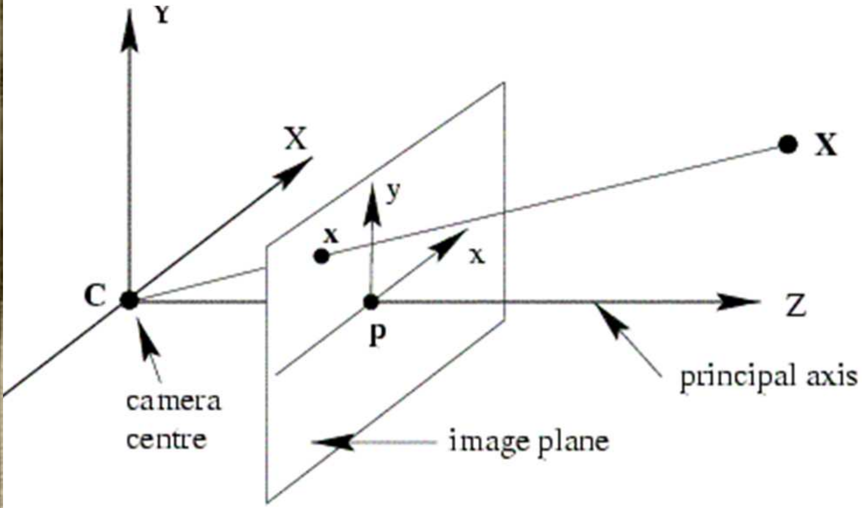
Summary

Various things we can model

1. Cameras
2. Radiometry and reflectance equation
3. BRDF – surface reflectance
   Lambertian BRDF
4. Light representation –
5. Image cues: shading, shadows, interreflections
6. Recovering Light (Inverse Light)

1. Projective camera model


[Dürer]



$\mathbf{x} = P\mathbf{X}$ $\quad P:3\times4$ **Projective Camera matrix**

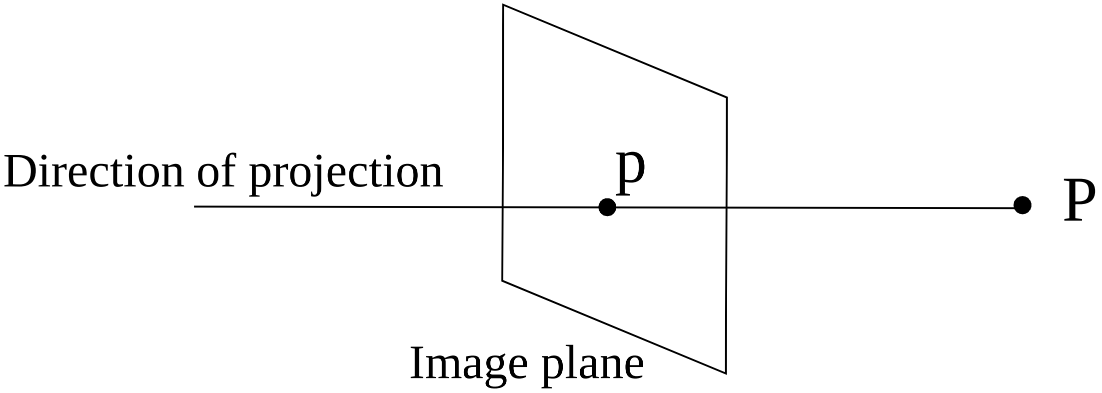
$\mathbf{x} = K\left[R \quad \mathbf{t}\right]\mathbf{X}$ $\quad K:3\times3$ **Eculicean Camera matrix**

$R, \mathbf{t}$ **Rotation, translation (ext. params)**

Orthographic camera model

Infinite Projection matrix - last row is (0,0,0,1)

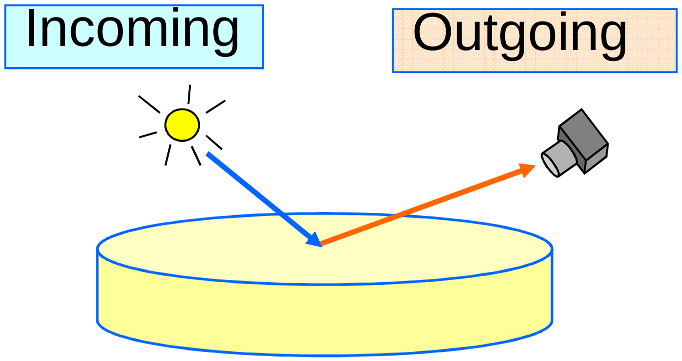
Good Approximations – object is far from the camera (relative to its size)



$$P_{orth} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Radiometry

- Foreshortening and Solid angle
- Measuring light : radiance
- Light at surface : interaction between light and surface
    - irradiance = light arriving at surface
    - BRDF
    - outgoing radiance

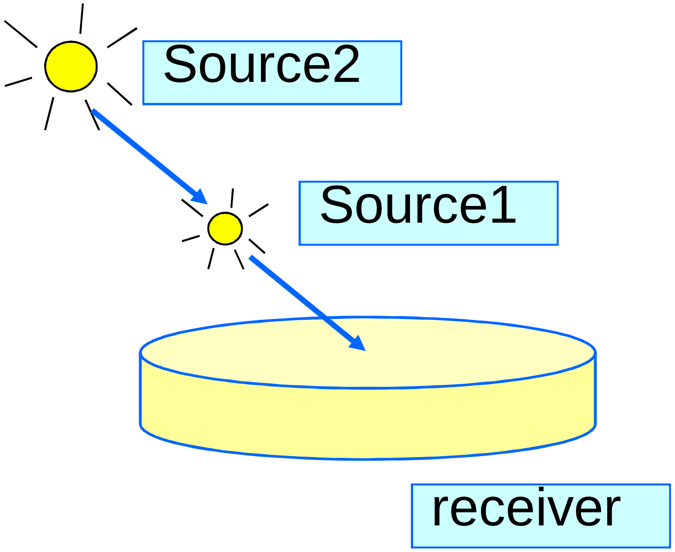


- Special cases and simplifications : Lambertain, specular, parametric and non-parametric models

Geometry and Foreshortening

Two sources that look the same to a receiver must have same effect on the receiver;

Two receivers that look the same to a source must receive the same energy.

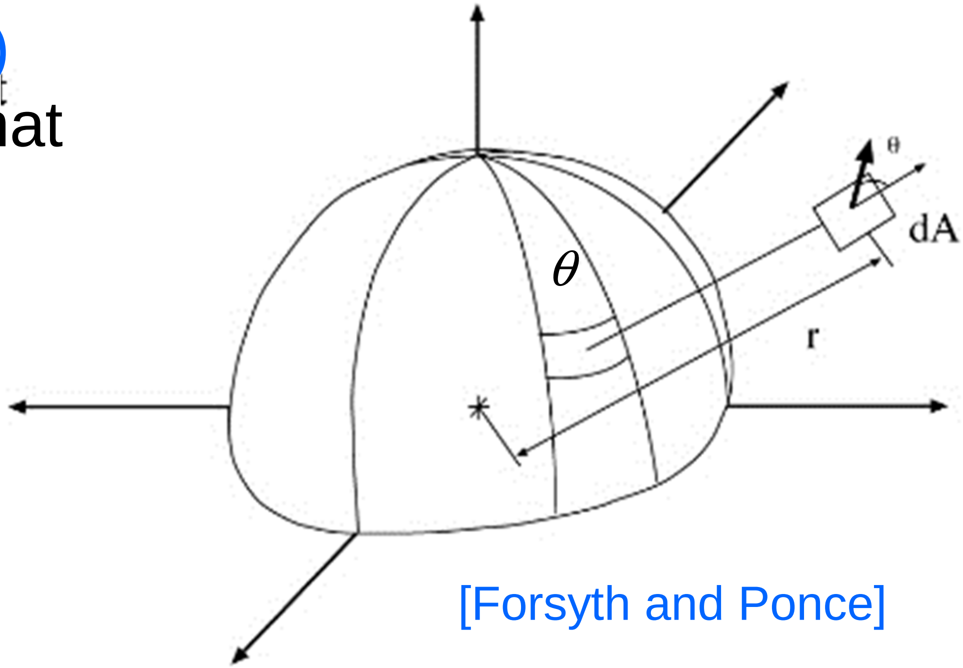


Solid angle

The solid angle subtended by a region at a point is the area projected on a unit sphere centered at the point

- Measured in steradians (sr)
- Foreshortening : patches that look the same, same solid angle.

$$d\omega = \frac{dA \cos\theta_n}{r^2}$$



[Forsyth and Ponce]

Integration inf in spherical coord:

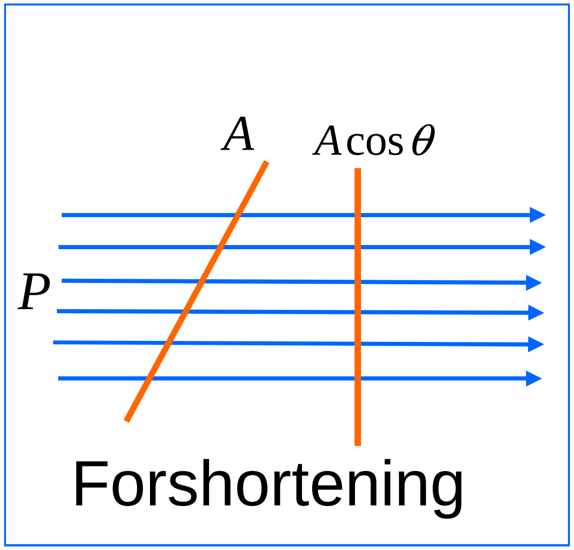
$$d\omega = \sin\theta \, d\theta \, d\phi$$

Radiance – emitted light

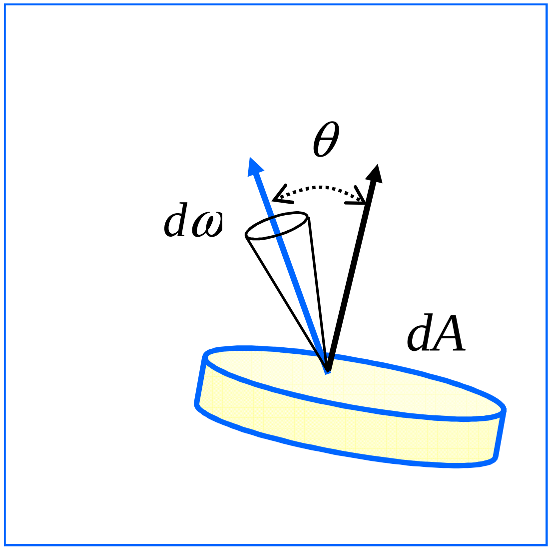
Radiance = power traveling at some point in a direction per unit area perp to direction of travel, per solid angle

- unit = watts/(m^{2}sr)
- constant along a ray

$$L(\mathbf{x},\theta,\phi)=\frac{P}{(dA\cos\theta)d\omega}$$



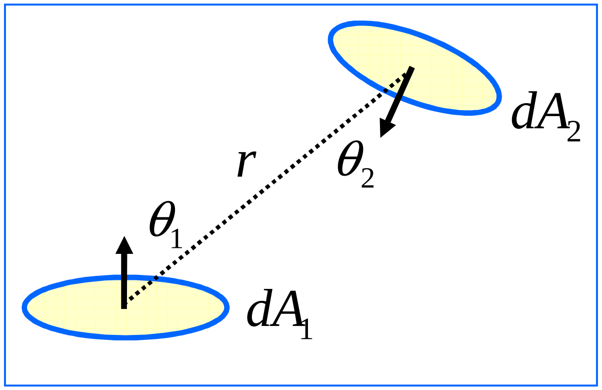
Forshortening



Radiance transfer :

Power received at dA$_2$ at dist r from emitting area dA$_1$

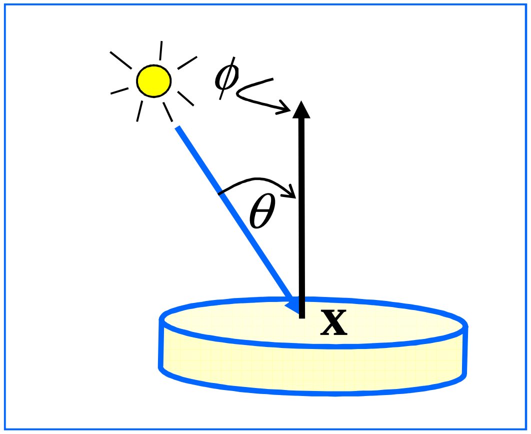
$$P_{1\to2}=LdA_1\cos\theta_1\underbrace{\boxed{\left(\frac{dA_2\cos\theta_2}{r^2}\right)}}_{d\omega_{21}}\qquad P_{1\to2}=P_{2\to1}$$



Light at surface : irradiance

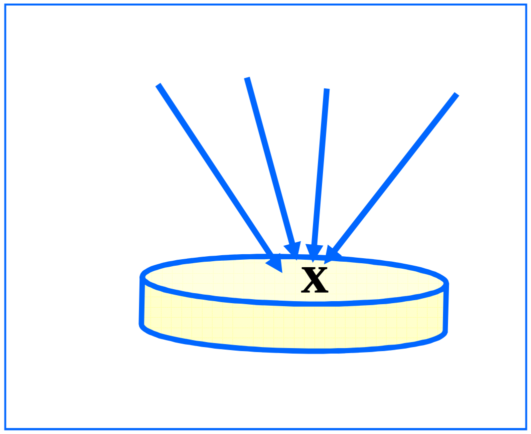
Irradiance = unit for light arriving at the surface

$$dE(\mathbf{x}) = L(\mathbf{x},\theta,\phi)\cos\theta d\omega$$

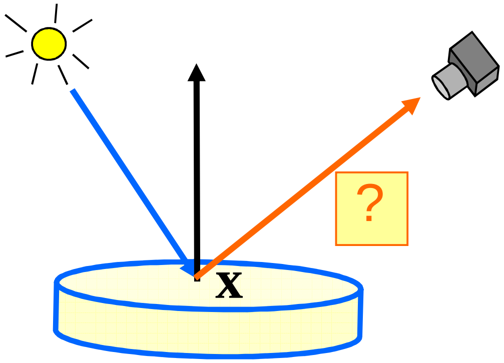


Total power = integrate irradiance over all incoming angles

$$E(\mathbf{x}) = \int_0^{2\pi}\int_0^{\pi/2} L(\mathbf{x},\theta,\phi)\cos\theta \underset{d\omega}{\boxed{\sin\theta d\theta d\phi}}$$



Light leaving the surface and BRDF



many effects :

- transmitted - glass
- reflected - mirror
- scattered – marble, skin
- travel along a surface, leave some other
- absorbed - sweaty skin

Assume:

- surfaces don't fluorescent
- cool surfaces
- light leaving a surface due to light arriving

BRDF = Bi-directional reflectance distribution function

Measures, for a given wavelength, the fraction of incoming irradiance from a direction ω_i in the outgoing direction ω_o [Nicodemus 70]

$$\rho(\mathbf{x},\theta_i,\phi_i,\theta_o,\phi_o)=\frac{L_o(\mathbf{x},\theta_o,\phi_o)}{L_i(\mathbf{x},\theta_i,\phi_i)\cos\theta_i d\omega}$$

Reflectance equation : measured radiance
(radiosity = power/unit area leaving surface

$$L_o(\mathbf{x},\theta_o,\phi_o)=\int_\Omega \rho(\mathbf{x},\theta_i,\phi_i,\theta_o,\phi_o)L(\theta_i,\phi_i)\cos(\theta_i)d\omega_i$$

Radiosity - summary

Radiance	Light energy along a ray	$L(\theta,\phi)=\frac{P}{(dA\cos\theta)d\omega}$
Irradiance	Unit incoming light	$dE(\mathbf{x})=L(\mathbf{x},\theta,\phi)\cos\theta d\omega$
Total Energy incoming	Energy at surface	$E_i(\mathbf{x})=\int_{\omega}L(\mathbf{x},\theta,\phi)\cos\theta d\omega$
Radiosity	Unit outgoing radiance	$L_o(\mathbf{x},\theta_o,\phi_o)=\int_{\Omega}\rho(\mathbf{x},\theta_i,\phi_i,\theta_o,\phi_o)L(\theta_i,\phi_i)\cos(\theta_i)d\omega_i$
Total energy leaving	Energy leaving the surface	$E_o=\int_{\Omega_o}\left[\int_{\Omega_i}\rho(\mathbf{x},\theta_i,\phi_i,\theta_o,\phi_o)L(\theta_i,\phi_i)\cos(\theta_i)d\omega_i\right]\cos(\theta_o)d\omega_o$

Example: Sunlight 1kW/m^2 . Artificial light <1/10th

3. BRDF properties

BRDF = Bi-directional reflectance distribution function

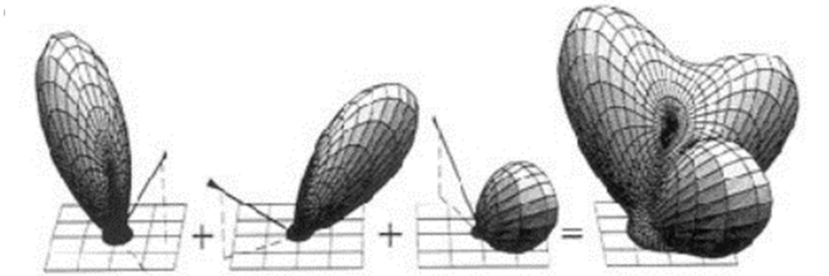
Measures, for a given wavelength, the fraction of incoming irradiance from a direction ω_i in the outgoing direction ω_o [Nicodemus 70]

Properties :

- Non-negative
- Helmholtz reciprocity
- Linear

$$\boxed{\rho(\theta_i,\phi_i,\theta_o,\phi_o) \geq 0}$$

$$\boxed{\rho(\theta_i,\phi_i,\theta_o,\phi_o) = \rho(\theta_o,\phi_o,\theta_i,\phi_i)}$$



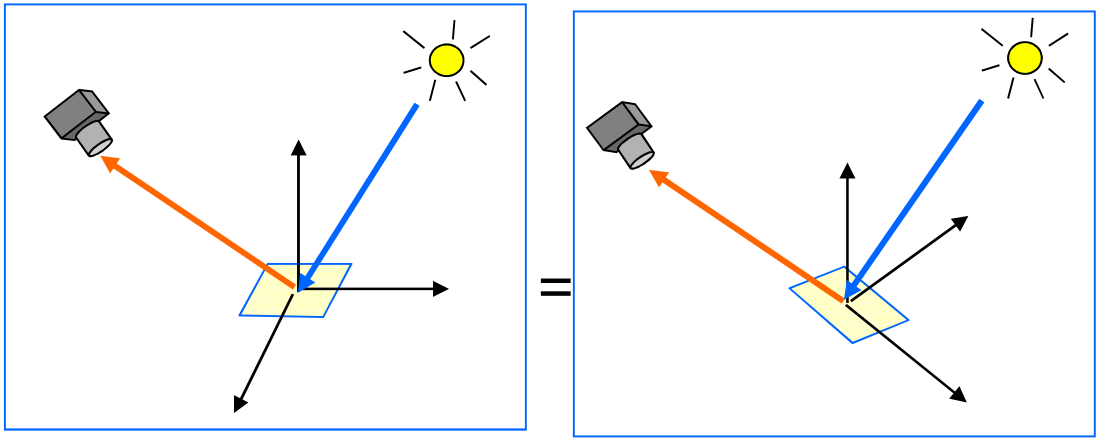
From Sillion, Arvo, Westin, Greenberg

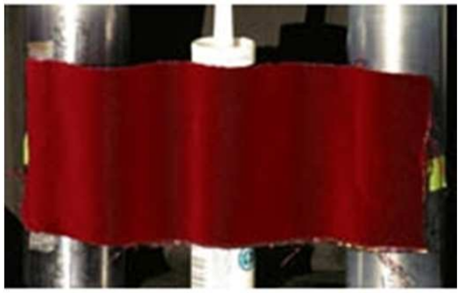
- Total energy leaving a surface less than total energy arriving at surface

$$\boxed{\int_{\Omega_i} L(\theta_i,\phi_i)\cos(\theta_i)d\omega_i \geq \int_{\Omega_o}\left[\int_{\Omega_i}\rho(\mathbf{x},\theta_i,\phi_i,\theta_o,\phi_o)L(\theta_i,\phi_i)\cos(\theta_i)d\omega_i\right]\cos(\theta_o)d\omega_o}$$

BRDF properties

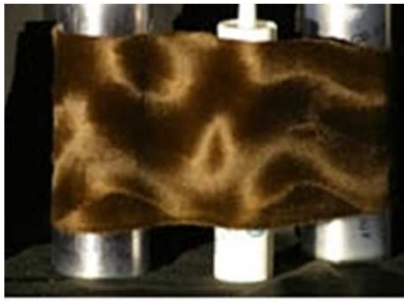
isotropic (3DOF)





$$\rho(\theta_i,\phi_i,\theta_o,\phi_o)=\rho(\theta_i,\theta_o,\phi_i-\phi_o)$$

anisotropic (4 DOF)



[Hertzmann&Seitz CVPR03]

Lambertian BRDF

- Emitted radiance constant/equal in all directions
- Models – perfect diffuse surfaces : clay, mate paper, …
- BRDF = constant = albedo
- One light source = dot product normal and light direction

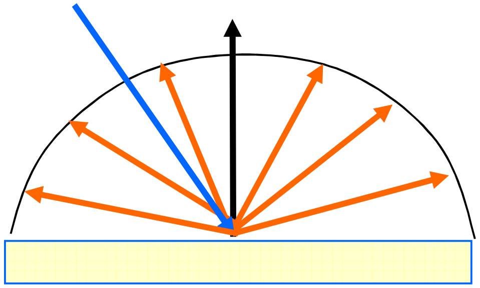
$$L_o(\mathbf{x}) = \rho L_i(\mathbf{x},\theta_i,\phi_i)\cos\theta_{i}^{\theta_i,\phi}$$

$$= \rho(\mathbf{N} \bullet \mathbf{L}_i)$$

albedo

normal

light dir



Diffuse reflectance acts like a low pass filter on the incident illumination.

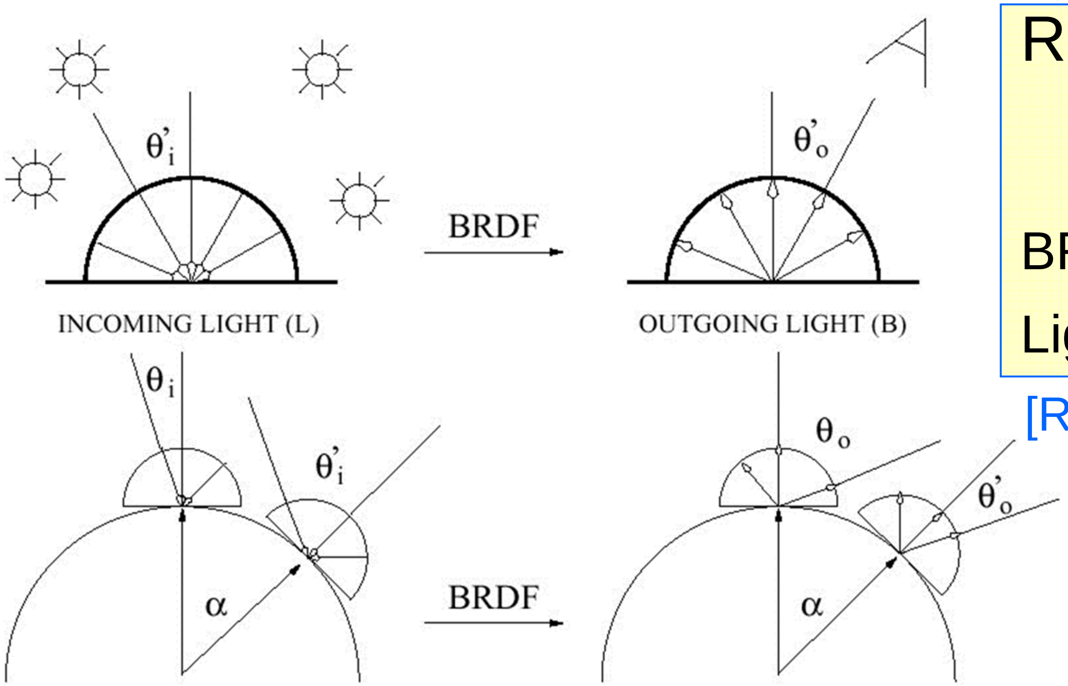
$$L_o(\mathbf{x},\theta_o,\phi_o) = \int_{\Omega'} \rho L(\theta_i,\phi_i)\cos(\theta_i)d\omega_i$$

Reflection as convolution

Reflectance equation

$$L_o(\mathbf{x},\theta_o,\phi_o) = \int_{\Omega'} \rho(\mathbf{x},\theta_i',\phi_i',\theta_o',\phi_o')L(\theta_i,\phi_i)\cos(\theta_i)d\omega_i$$

$$= \int_{\Omega} \rho(\mathbf{x},\theta_i',\phi_i',\theta_o',\phi_o')L(R_{\alpha,\beta}(\theta_i',\phi_i'))\cos(\theta_i)d\omega_i$$



Reflection behaves like a convolution in the angular domain

BRDF – filter

Light - signal

[Ramamoorthi and Hanharan]

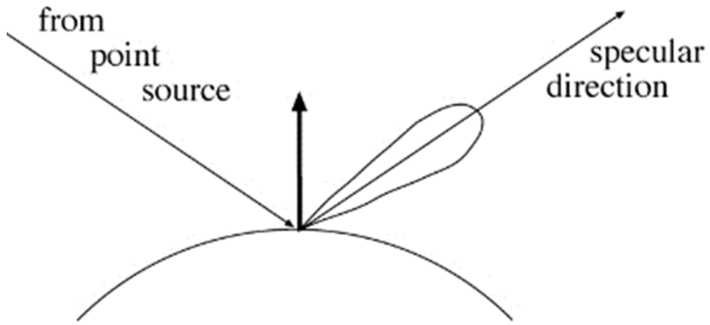
Specular reflection

Smooth specular surfaces

- Mirror like surfaces
- Light reflected along specular direction
- Some part absorbed

Rough specular surfaces

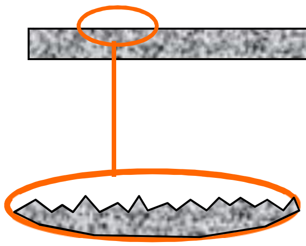
- Lobe of directions around the specular direction
- Microfacets



Lobe

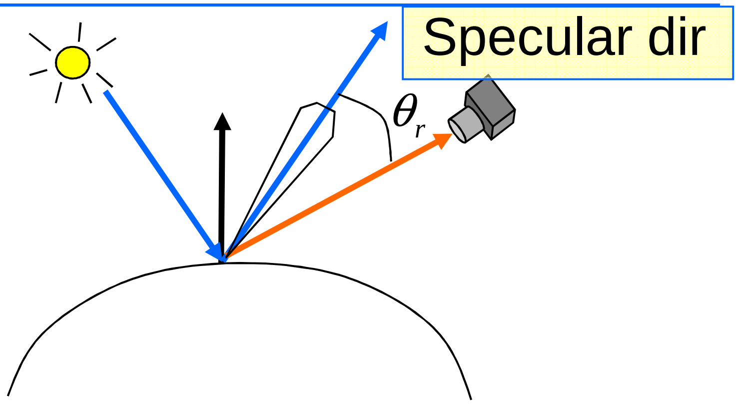
- Very small – mirror
- Small – blurry mirror
- Bigger – see only light sources
- Very big – fait specularities

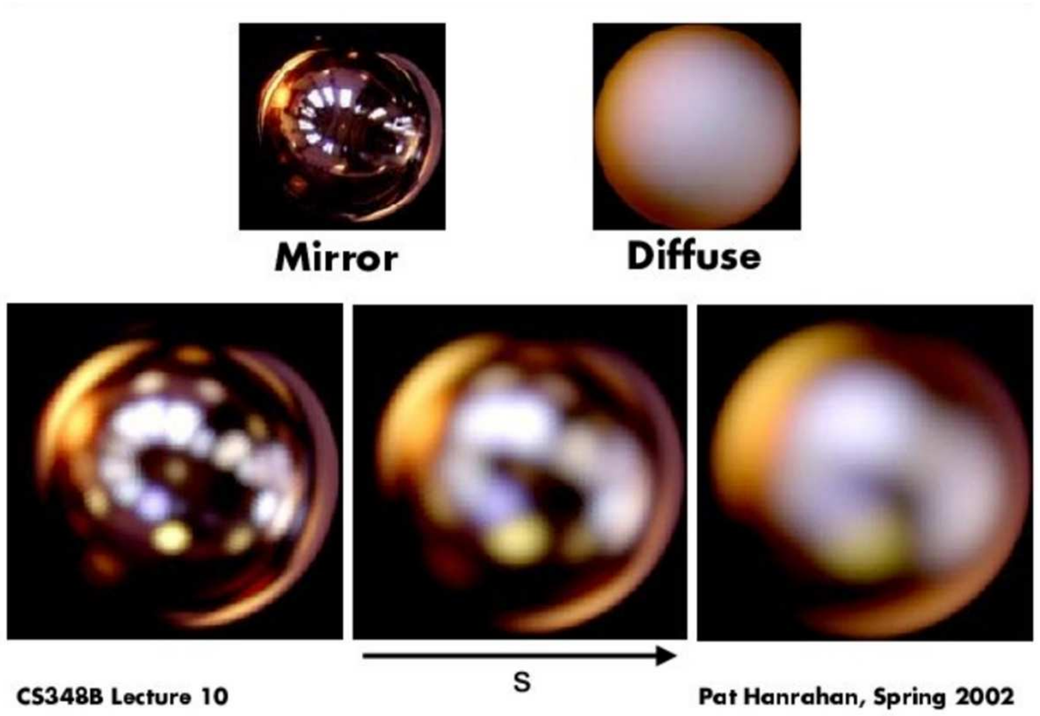
Phong model



Symmetric V shaped microfacets

$$\rho_{Phong} = k_d + k_s \frac{(\cos\theta_r)^n}{\cos\theta_i}$$





Modeling BRDF

Parametric model:

- Lambertian, Phong
- Physically based:
  - Specular [Blinn 1977] [Cook-Torrace 1982][Ward 1992]
  - Diffuse [Hanharan, Kreuger 1993]
  - Generalized Lambertian [Oren, Nayar 1995]
  - Throughly Pitted surfaces [Koenderink et al 1999]
- Phenomenological:
  - [Koenderink, Van Doorn 1996]

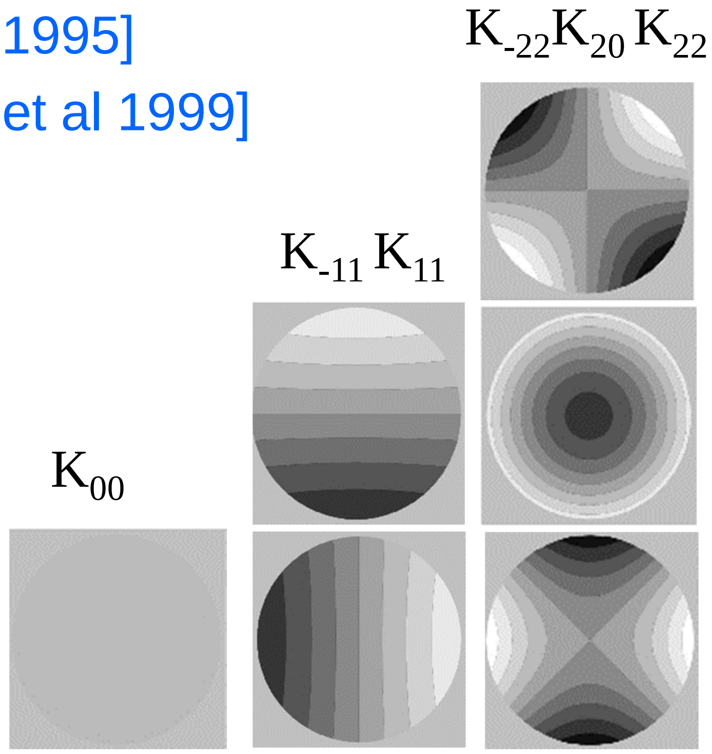
    summarize empirical data

    orthonormal functions on the $\mathbf{H}_{S^2} \times \mathbf{H}_{S^2}$

    ($\mathbf{H}_{S^2}$ hemisphere)

    same topol. as unit disk

    (Zernike Polynomials)

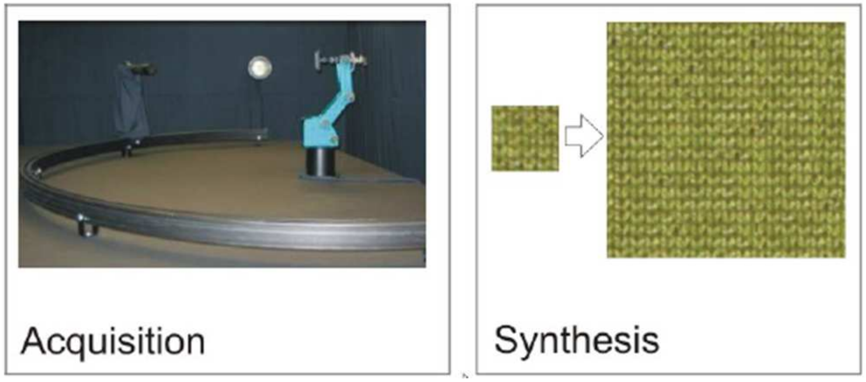


Measuring BRDF

Gonioreflectometers

- Anisotropic 4 DOF
- Non-uniform

BTF [Dana et al 1999]



[Müller 04]

More than BRDF – BSSRDF

(bidirectional surface scattering distribution)

BRDF

BSSRDF

[Jensen, Marschner, Leveoy, Hanharan 01]

Do SFS from here.

4. Light representations

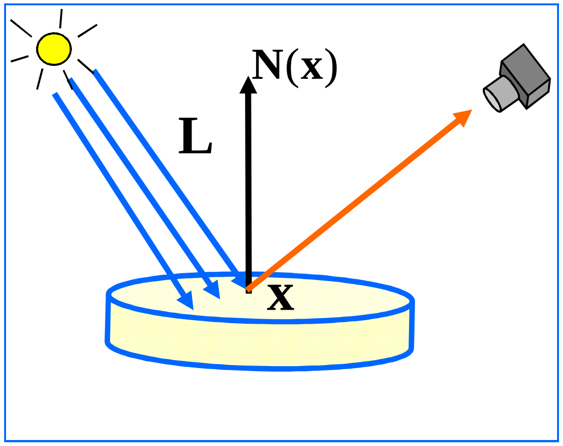
Light source –

theoretical framework [Langer, Zucker-What is a light source]

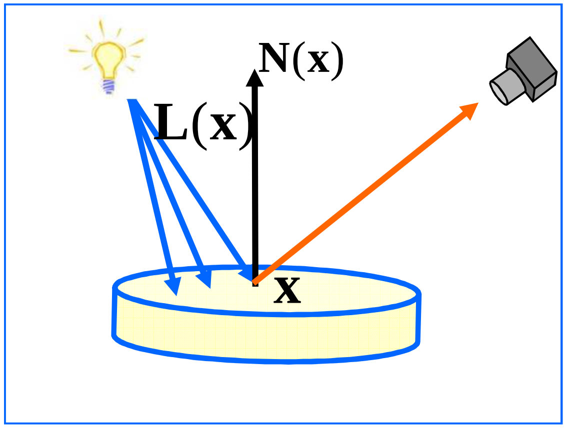
Point light sources

- Infinite

$$L_o(\mathbf{x}) = \rho(\mathbf{x})E\cos\theta_i = \rho(\mathbf{x})\mathbf{N}(\mathbf{x})\bullet\mathbf{L}$$



- Nearby



$$L_o(\mathbf{x}) = \rho(\mathbf{x})\frac{E\cos\theta_i(\mathbf{x})}{r^2} = \rho(\mathbf{x})\frac{\mathbf{N}(\mathbf{x})\bullet\mathbf{L}(\mathbf{x})}{r^2}$$

Choosing a model

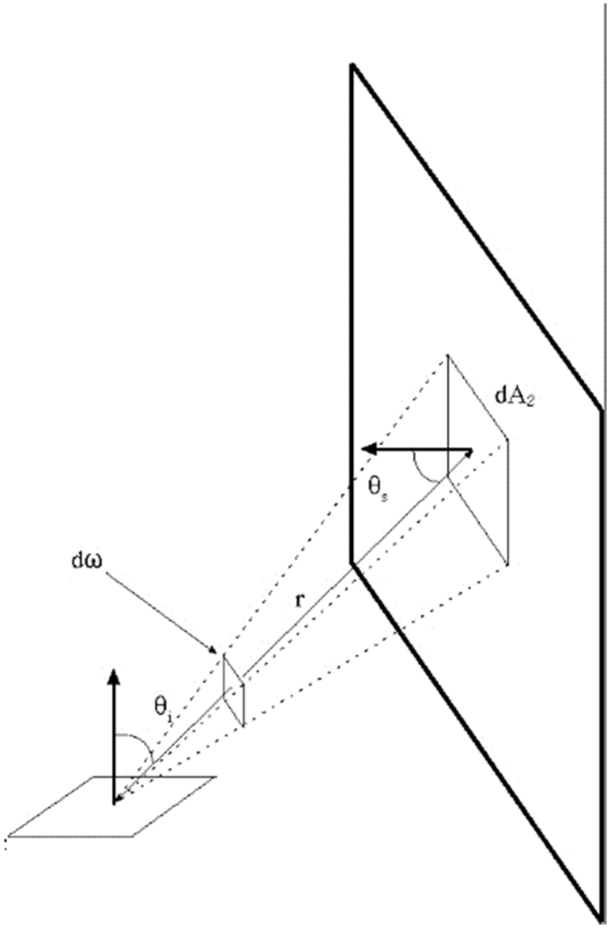
- infinite - sun
- finite - distance to source is similar in magnitude with object size and distance between objects
  - indoor lights

Area sources

Examples : white walls, diffuse boxes

Radiosity : adding up contributions over the section of the view hemisphere subtended by the source

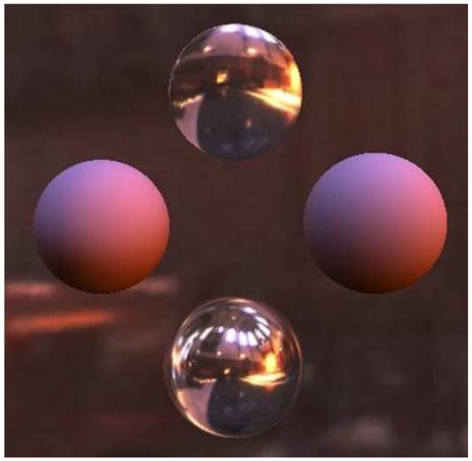
$$L_o(x) = \rho(x) \int_{source} E(Q) \frac{\cos\theta_i \cos\theta_s}{\pi r^2} dA_Q$$



Enviromental map

Illumination hemisphere
Large number of infinite point light sources

[Debevec]



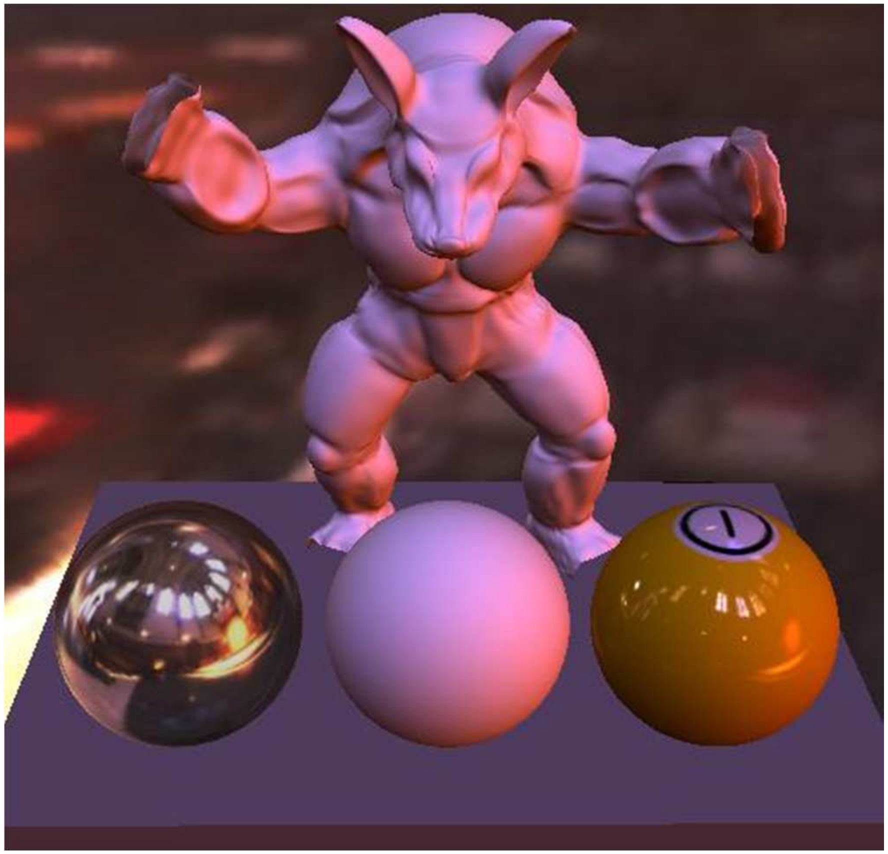
5. Image cues

shading, shadows, specularities …

Shading

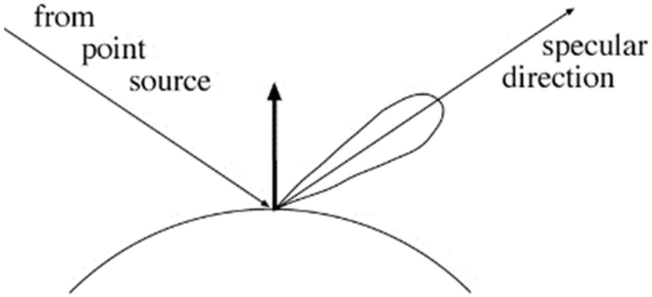
Lambertian reflectance $L_o(\mathbf{x}) = \rho L \boxed{\cos\theta} = \rho L \boxed{(\mathbf{N} \bullet \mathbf{L}_i)}$

Shading = observed <u>smooth</u> color variation due to <u>Lambertian</u> reflectance



Specular highlights

High frequency changes in observed radiance due to general BRDF (shiny material)



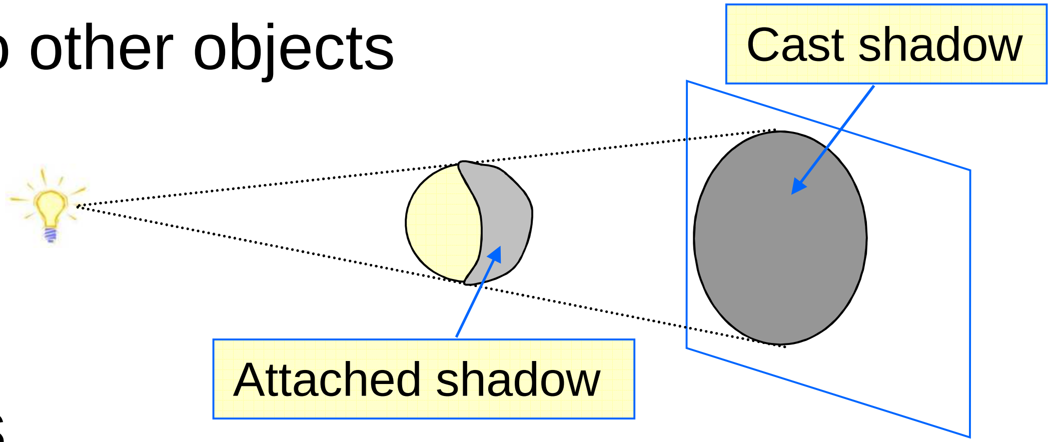
Shadows (local)

1. Point light sources

Points that cannot see the source – modeled by a visibility binary value

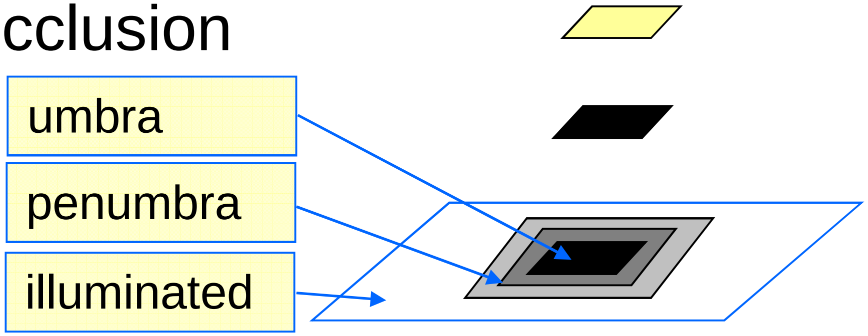
attached shadows = due to object geometry (self-shadows)

cast shadows = due to other objects



2. Area light sources

Soft shadows – partial occlusion



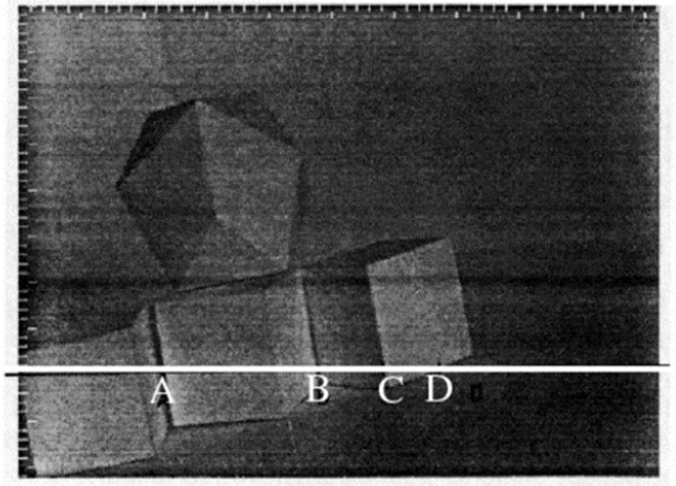
Interreflections

Local shading – radiosity only due to light sources

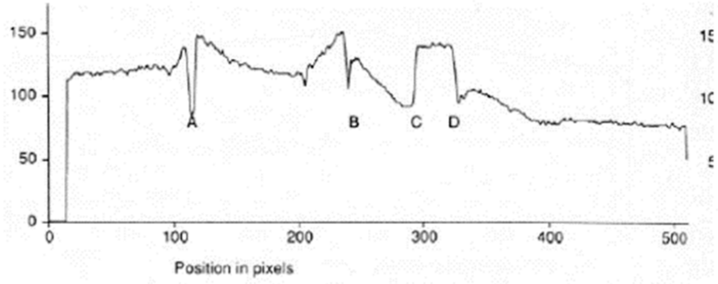
[computer vision, real-time graphics]

Global illumination – radiosity due to radiance reflected from light sources as well as surfaces

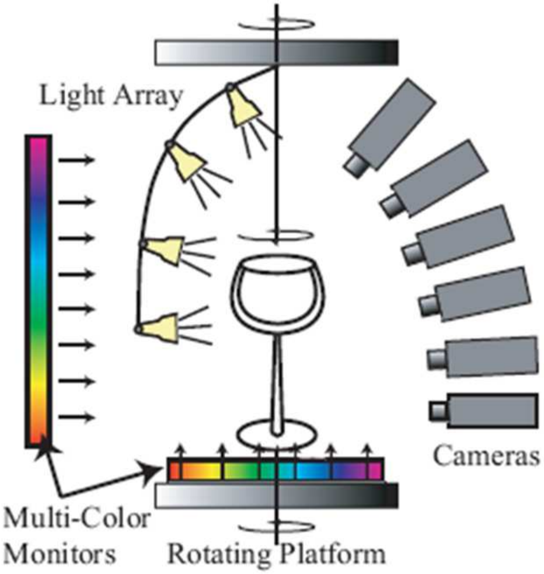
[computer graphics]



White room under bright light.
Below cross-section of image intensity
[Forsyth, Zisserman CVPR89]



Transparency



Special setups for image aquisition

Enviroment mating

[Matusik et al Eurographics 2002]

[Szeliski et al Siggraph 2000]

6. Inverse light

$$L_o(\mathbf{x},\theta_o,\phi_o)=\int_{\Omega'}\rho(\mathbf{x},\theta_i',\phi_i',\theta_o',\phi_o')\boxed{L(\theta_i,\phi_i)}\cos(\theta_i)d\omega_i$$

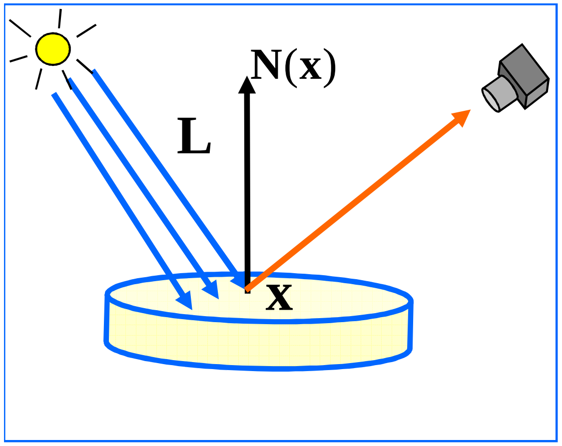
Deconvolution of light from observed radiance

Assumptions:

- known camera position
- known object geometry
- [known or constant BRDF]
- [uniform or given texture]

Estimating multiple point light sources
Estimating complex light : light basis

Estimating point light sources



Lambertian reflectance – light from shading

Infinite single light source

$$L_o(\mathbf{x}) = \rho(\mathbf{x}) L \cos\theta \;\; = \rho L(\mathbf{N}(\mathbf{x}) \bullet \mathbf{L})$$

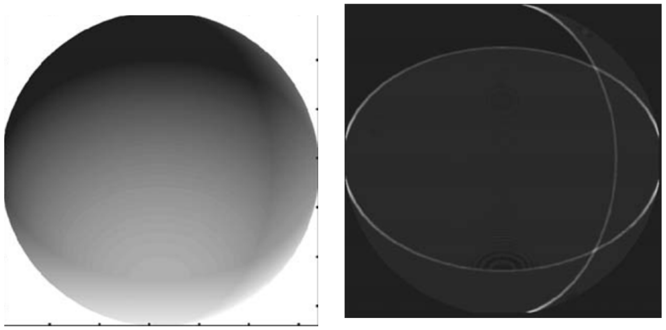
- known or constant albedo ρ
- known $\mathbf{N(x)}$
- recover L (light color) and $\mathbf{L}$ (direction) from >= 4 points.

Multiple light sources

Calibration sphere
Critical points/curves
- Sensitive to noise

[Yang Yuille 91]

[Bouganis 03]



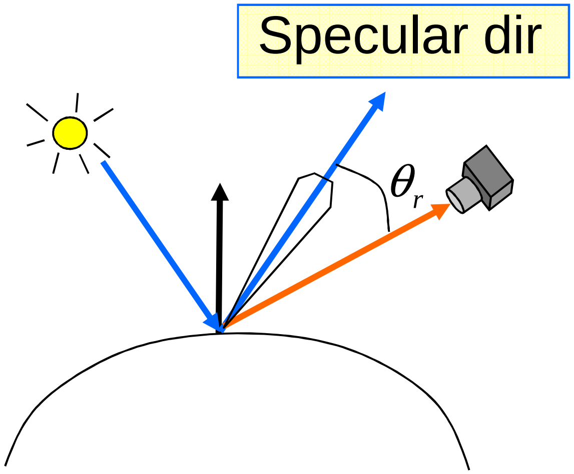
Estimating complex light

Diffuse reflectance acts like a low pass filter on the incident illumination.

Can only recover low frequency components. Use other image cues !

Light from specular reflections

- Recover a discrete illumination hemisphere
- Specular highlights appear approximately at mirror directions between light and camera rays
- Trace back and compute intersection with hemisphere



Recovered hemisphere

Capture light direcly using a mirror sphere

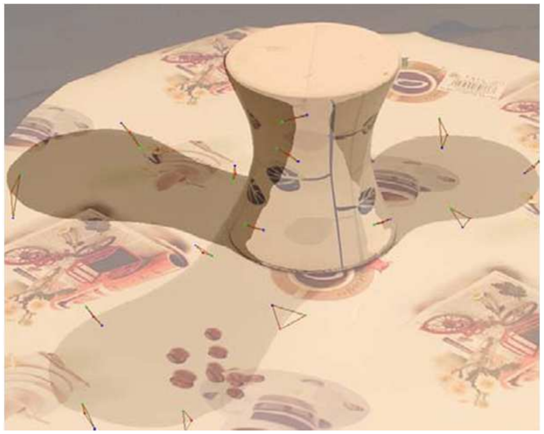
[Nishimo,Ikeuchi ICCV 2001]

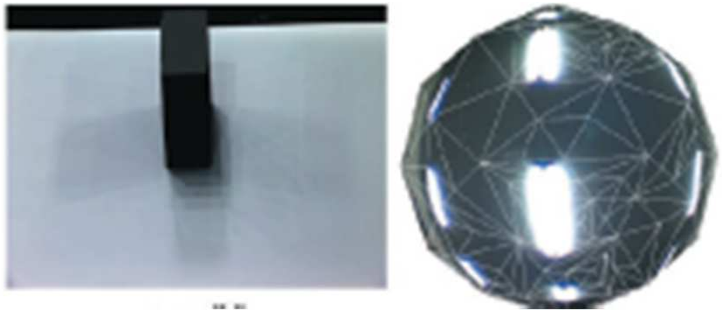
Estimating complex light

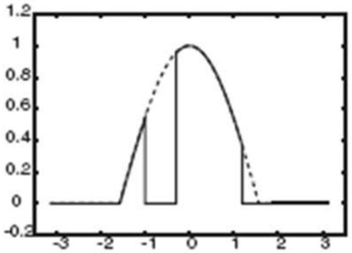
Light from cast shadows

[Li Lin Shun 03]

[Sato 03]







- Shadows are caused by light being occluded by the scene.
- The measured radiance has high frequency components introduced by the shadows.

$$L_o(\mathbf{x},\theta_o,\phi_o)=\int_{\Omega}\boxed{V(\mathbf{x},\theta_i,\phi)}\rho L(\theta_i,\phi_i)\cos(\theta_i)d\omega_i$$

Shadow indicator

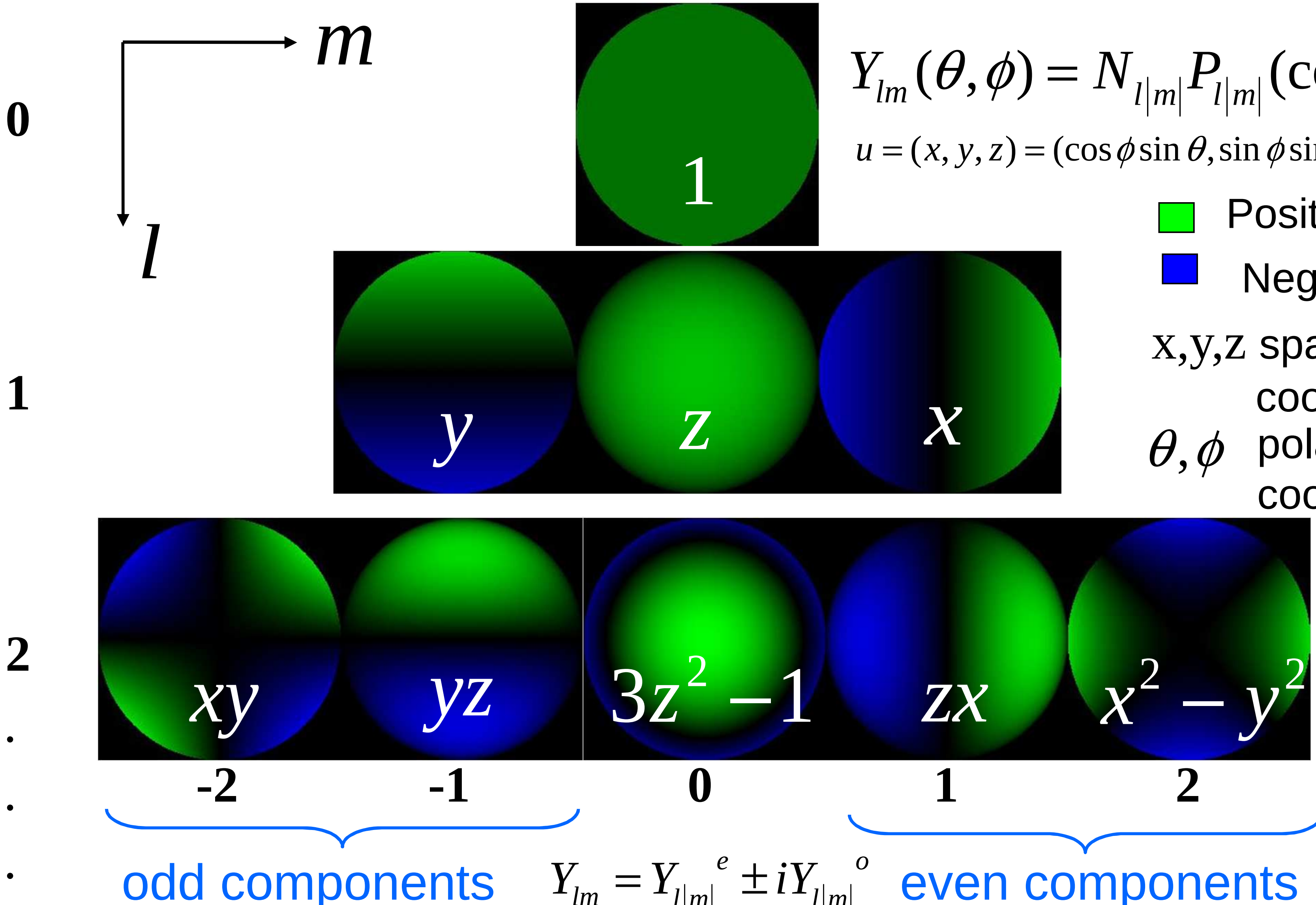
Light basis representation

Spherical harmonics basis

- Analog on the sphere to the Fourier basis on the line or circle
- Angular portion of the solution to Laplace equation in spherical coordinates
- Orthonormal basis for the set of all functions on the surface of the sphere $\nabla^2\psi = 0$

$$Y_{lm}(\theta,\phi) = \underbrace{\sqrt{\frac{(2l+1)}{4\pi}\frac{(l-|m|)!}{(l+|m|)!}}}_{\text{Normalization factor}}\underbrace{P_{l|m|}(\cos\theta)}_{\text{Legendre functions}}\underbrace{e^{im\phi}}_{\text{Fourier basis}}$$

Illustration of SH



$$Y_{lm}(\theta,\phi) = N_{l|m|}P_{l|m|}(\cos\theta)e^{im\phi}$$

$$u=(x,y,z)=(\cos\phi\sin\theta,\sin\phi\sin\theta,\cos\theta)$$

🟩 Positive

🟦 Negative

x,y,z space coordinates;

θ,ϕ polar coordinates

odd components $Y_{lm} = Y_{l|m|}^{e} \pm iY_{l|m|}^{o}$ even components

Properties of SH

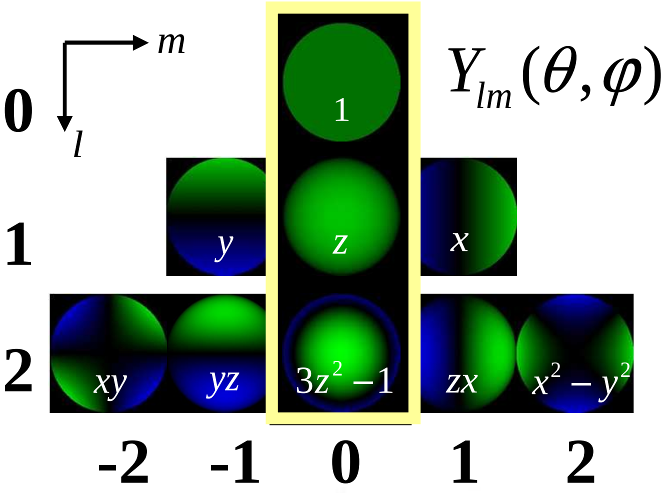
Function decomposition

f piecewise continuous function on the surface of the sphere

$$f(u)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l} f_{lm}Y_{lm}(u)$$

where

$$f_{lm}=\int_{S^2} f(u)Y^{*}{}_{lm}(u)du$$



Rotational convolution on the sphere

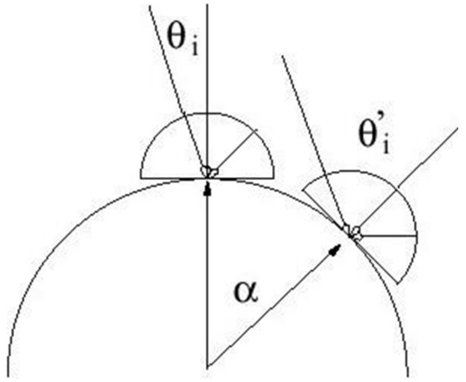
Funk-Hecke theorem:

k circularly symmetric bounded integrable function on [-1,1]

$$k(u)=\sum_{l=0}^{\infty} k_l Y_{l0}$$

$$k*Y_{lm}=\alpha_l Y_{lm} \qquad \alpha_l=\sqrt{\frac{4\pi}{2l+1}}k_l$$

convolution of a (circularly symmetric) function k with a spherical harmonic Y_{lm} results in the same harmonic, scaled by a scalar α_l.



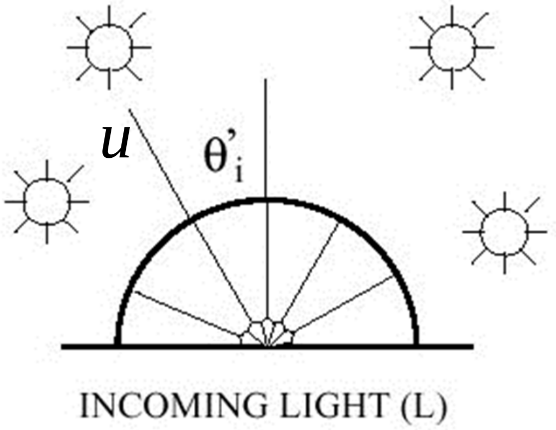
Reflectance as convolution

Lambertian reflectance

One light $$R(u') = l(u)\rho \max(0, u \bullet u')$$

Lambertian kernel $$k(u \bullet u') = \max(0, u \bullet u')$$

Integrated light $$R(u') = \int_{S^2} k(u \bullet u') l(u) du$$



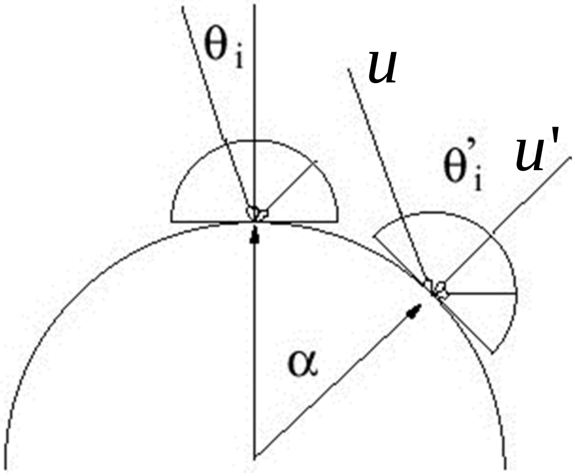
SH representation

light

$$l(u) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} l_{lm} Y_{lm}(u)$$

Lambertian kernel

$$k = \sum_{l=0}^{\infty} k_l Y_{l0}$$



Lambertian reflectance (convolution theorem)

$$R = k * l = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} (\sqrt{\frac{4\pi}{2l+1}} k_l l_{lm}) Y_{lm} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} r_{lm} Y_{lm}$$

Convolution kernel

Lambertian kernel

$$k(u \bullet u') = \max(0, u \bullet u')$$

$$k = \sum_{l=0}^{\infty} k_l Y_{l0}$$

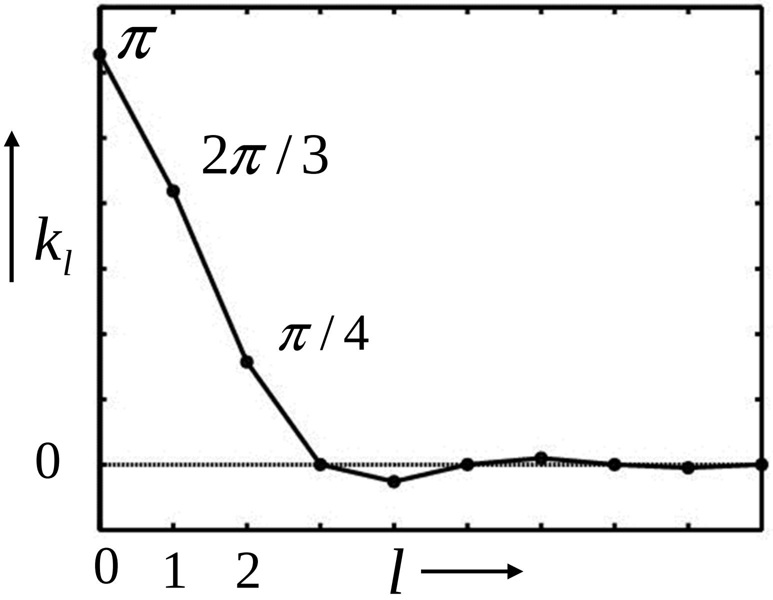
$$k_l = \begin{cases} \frac{\sqrt{\pi}}{2} & n = 0 \\ \frac{\sqrt{\pi}}{3} & n = 1 \\ (-1)^{l/2+1} \frac{\sqrt{(2l+1)\pi}}{2^l (l-1)(l+2)} \binom{l}{l/2} & n \geq 2, \text{even} \\ 0 & n \geq 2, \text{odd} \end{cases}$$

Asymptotic behavior of k_l for large l

$k_l \approx l^{-2}$ $r_{lm} \approx l^{-5/2}$

- Second order approximation accounts for 99% variability
- k like a low-pass filter

[Basri & Jacobs 01]
[Ramamoorthi & Hanrahan 01]



From reflectance to images

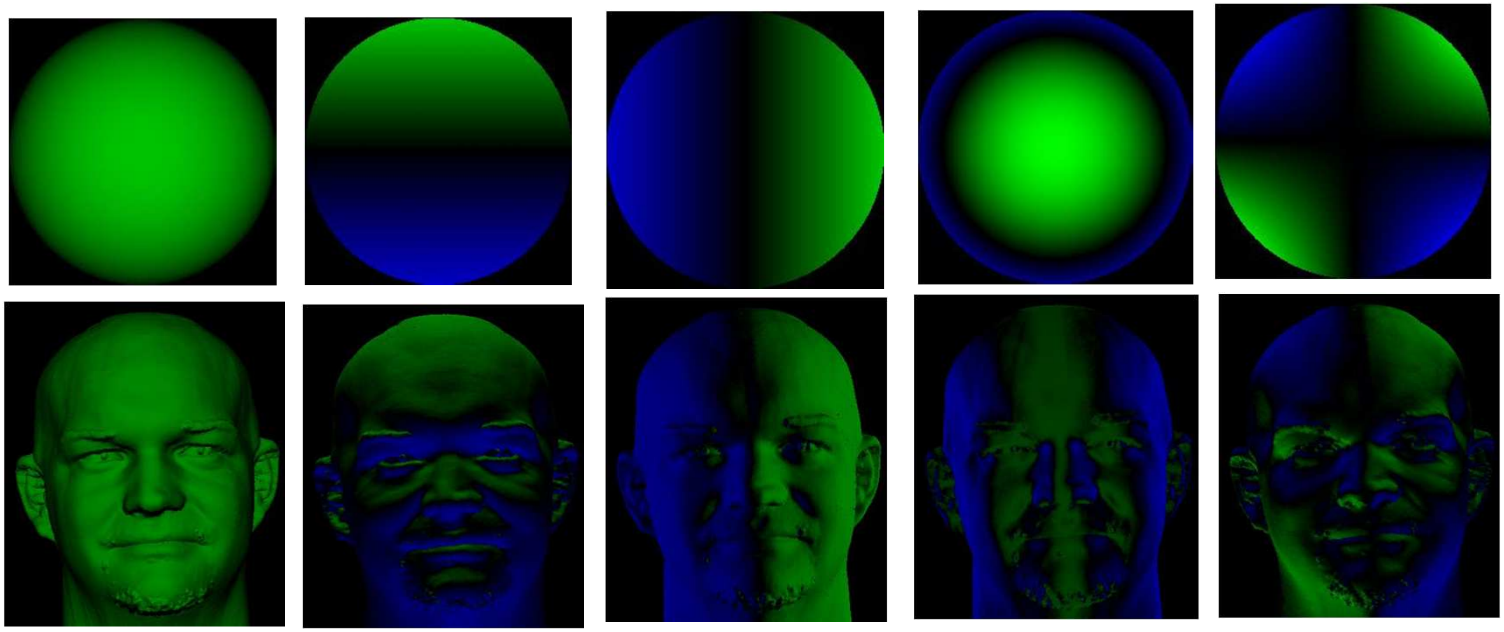
Unit sphere $\Rightarrow$ general shape
Rearrange normals on the sphere

Reflectance on a sphere

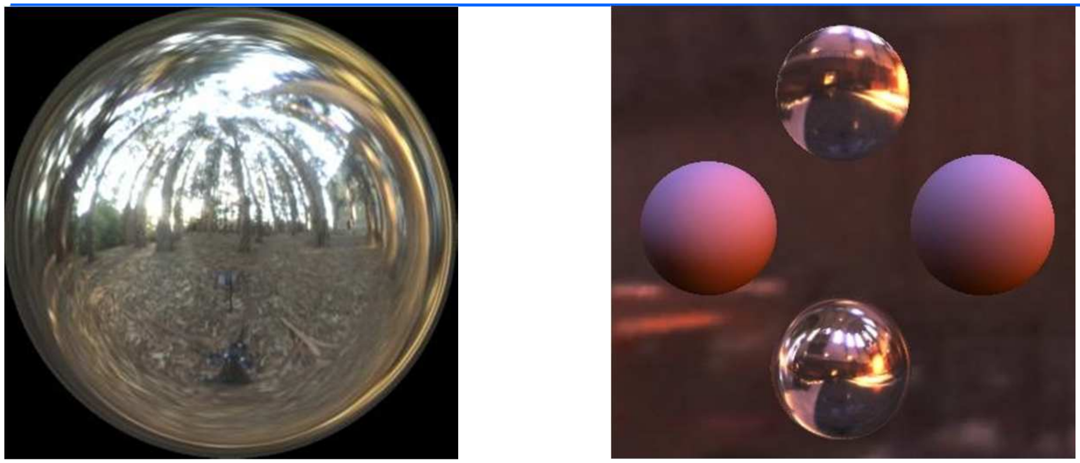
$$R = k * l = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} r_{lm} Y_{lm}$$

Image point with normal n_i

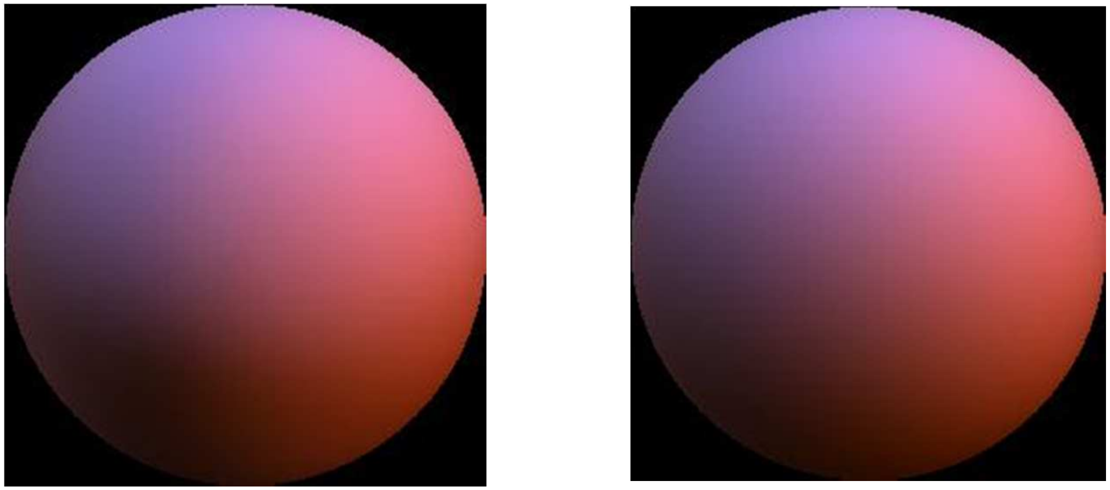
$$I_i = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \rho_i r_{lm} Y_{lm}(n_i)$$



Example of approximation



Exact image 9 terms approximation



<u>Efficient rendering</u>

- known shape
- complex illumination (compressed)

[Ramamoorthi and Hanharan: An efficient representation for irradiance enviromental map Siggraph 01]

Extensions to other basis

SH light basis limitations:

- Not good representation for high frequency (sharp) effects ! (specularities)
- Can efficiently represent illumination distribution localized in the frequency domain
- BUT a large number of basis functions are required for representing illumination localized in the angular domain.

Basis that has both frequency and spatial support

⇨ Wavelets	[Upright CRV 07] [Okabe Sato CVPR 2004]
⇨ Spherical distributions	[Hara, Ikeuchi ICCV 05]
⇨ Light probe sampling	[Debevec Siggraph 2005] [Madsen et al. Eurographics 2003]

Basis with local support

Median cut

[Debevec
Siggraph 2005]

Not localized
in frequency!

Wavelet Basis

[Upright Cobzas 07]