We propose a simple local minimization algorithm for

$$\min_{\|x\|_2=1} \|Ax\|_1 \quad , \quad \text{where } \|x\|_1 \triangleq \sum_i |x_i| \; , \quad \|x\|_2 \triangleq \sqrt{\sum_i |x_i|^2}$$

The algorithm will be based on the following trivial observation:

$$2|\alpha| = \min_{\eta \geq 0} \frac{|\alpha|^2}{\eta} + \eta$$

apparently $\displaystyle \min_{\|x\|_2=1} \|Ax\|_1 = \min_{\|x\|_2=1} \sum_i |a_i x|$, where $a_i = A_{i:}$, the i-th row of A

$$= \frac{1}{2} \min_{\|x\|_2=1} \min_{\eta \geq 0} \sum_i \frac{1}{\eta_i} \cdot |a_i x|^2 + \eta_i$$

$$= \frac{1}{2} \min_{\eta \geq 0} \min_{\|x\|_2=1} 1^T \eta + \left\| \mathrm{Diag}\left(\frac{1}{\sqrt{\eta}}\right) \cdot A \cdot x \right\|_2^2 ,$$

where $1 = (1, 1, \cdots, 1)^T$

$\sqrt{\eta} = (\sqrt{\eta_1}, \sqrt{\eta_2}, \cdots, \sqrt{\eta_k})^T$

$\mathrm{Diag}(\sqrt{\eta})$ is the diagonal matrix with $\sqrt{\eta}$ on the diagonal

Observation: ① given η, solve x reduces to the familiar DLT algorithm (with weights η_i)

i.e., x is given by DLT with A replaced by $\mathrm{Diag}\left(\frac{1}{\sqrt{\eta}}\right) \cdot A$

② given x, solve η is trivial:

$$\eta_i = |a_i x|$$

③ we just alternate between ① & ② until some convergence criteria is met (for instance, when the objective value does not seem to decrease much).

This simple algorithm is an instance of coordinate descent.

(warning: we have $\frac{1}{\sqrt{\eta_i}}$ in the algorithm, when $\eta_i \to 0$, $\frac{1}{\sqrt{\eta_i}}$ blows up and could cause your implementation to fail. Handle this appropriately.) Note: the weight $\frac{1}{\sqrt{\eta_i}}$ indicates how "normal" a_i is.