CMPUT 474 - Midterm Exam (25%)

Instructor: *E. Elmallah*

Date: March 7, 2000

Questions: 4	**Total Pages:** 2
Time: 70 minutes	**Calculators:** allowed
Closed Book	(but provide no advantage)

- ***No questions during exam time.***
- ***If you are unsure, write down your assumptions.***
- ***Answer each question on a separate paper leaf, in order.***

Question 1: `[25 marks]`

Answer **true** or **false**, and give a ***brief explanation***.

1. If L is an infinite language over an alphabet Σ then $\overline{L}$ (the complement of L with respect to Σ^*) is **finite**.

2. If L is a regular language then its complement $\overline{L}$ is also a regular language.

3. If r and s are regular expressions then $(r+s)^* = (r+sr^*)^*$.

4. To show that a language L is **not** regular, it suffices to show that some subset of L is **not** regular.

5. If M is an NFA on n states then M is equivalent to some DFA M' with $O(n^2)$ states.

6. Every DFA can be converted to an equivalent DFA that has a single accept state.

7. Every NFA can be converted to an equivalent NFA that has a single accept state.

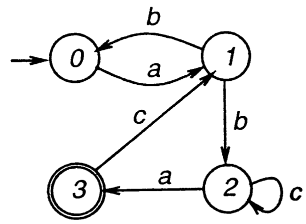
8. If $L = \{w|\ w \text{ is the empty string } \epsilon, \text{ or } w \text{ ends in a } 0\}$ then $p = 4$ is the ***minimum*** value that works as a pumping *length* for L. (Recall, the pumping length is the constant used by the Pumping lemma for regular languages.)

Question 2: `[22 marks]`

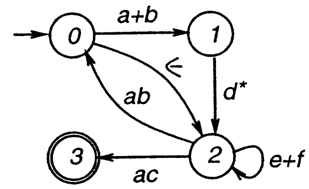
G₁

$S \to AB \mid C$
$A \to pAqAr \mid C \mid \epsilon$
$B \to BC \mid t$
$C \to u \mid v$

M₁



M₂



G₂

$S \to AB \mid C$
$A \to pAqAr \mid C$
$B \to BC \mid t$
$C \to u \mid v$

1. Give an equivalent CFG to G_1 with no ϵ-rules.
2. Give an equivalent CFG to G_2 with no unit-rules.
3. Draw a FA that recognizes L^* (the Kleene closure) for the language L recognized by M_1. Label all states, show the start state and all final states. Explain your answer.
4. For the generalized FA M_2, the symbols ϵ, and a through f indicate regular expressions. Show the state diagram (and all regular expressions) obtained after applying the algorithm that converts NFA to regular expressions to eliminate state 2.

Question 3: [10 marks]
Let L be a language on an alphabet Σ. Define the relation $\equiv_L$ on Σ^* as follows: for any arbitrary strings $x, y \in \Sigma^*$, $x \equiv_L y$ if and only if for every string $z \in \Sigma^*$, either ($xz \in L$ and $yz \in L$), or ($xz \notin L$ and $yz \notin L$). Note that $\equiv_L$ is an equivalence relation.

1. What are the equivalence classes of $\equiv_L$, where L is given by the regular expression 0^*10^*?
2. Show that if L is specified by some regular expression then $\equiv_L$ has a finite number of equivalence classes.

Question 4: [12 marks]

1. Write the pumping lemma.
2. Use the pumping lemma to prove that $L = \{ww \mid w \in \{a, b\}^*\}$ is not regular.

*** THE END ***