

CMPUT 474 - Final Exam (40%) E. Elmallah

Date: April 25, 2000

Questions: 5

Time: 120 minutes

Closed Book

Total Pages: 2

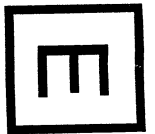
Calculators: allowed

(but provide no advantage)

- *No questions during exam time.*
- *If you are unsure, write down your assumptions.*
- *Answer each question on a separate paper leaf, in order.*

Question 1 [25 marks]. Give a mathematical definition of the following. Briefly explain your answer.

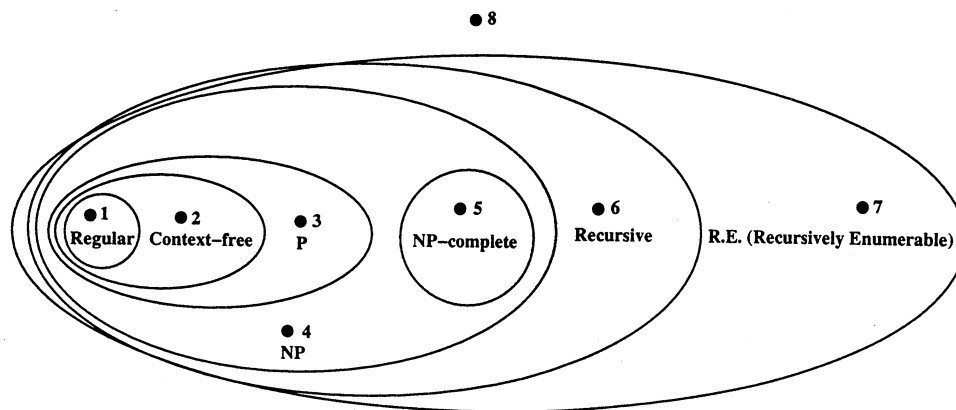
1. the transition function of a DTM $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$
2. the transition function of a nondeterministic TM (NTM)
3. the *instantaneous description* (i.e., *configuration*) of a Turing machine with one read-only input tape and two work tapes
4. for two languages A and B , the notion of A *reduces to* B (written $A \leq B$) suitable for proving *NP-hardness* results
5. a *correspondence* (i.e. a function that is one-to-one and onto) to show that the set of positive rational numbers $\{\frac{x}{y} \mid x \text{ and } y \text{ are positive integers}\}$ is *countable*.



04831
 CMPUT 474 (B1)
 ELMALLAH, E.
 APR 00 FINAL
 PAGES: 2

Question 2 [40 marks].

Put the following languages in the most appropriate place in the hierarchy. You have to specify the number of the point in the figure that suits best for the language. Briefly justify your answer.



1. $A = \{b^i c^i \mid i \geq 0\} \cup \{ab^i c^{2i} \mid i \geq 0\}$

2. $B = \{a^i b^j c^k \mid (i, j > 10) \text{ and } (k < 20)\}$.
3. $C = \{a^i b^k c^k \mid i, k > 0\} \cap \{a^i b^i c^k \mid i, k > 0\}$
4. $D = \{ \langle r \rangle \mid r \text{ is a regular expression written with the characters:}$
 $\text{"a" to "z", " ", "*", "+", "(", ")" } \}$
for example, $\langle (ab+c)^* + d^* \rangle \in D$, but $\langle ** (a**)^* \rangle \notin D$ (the former is a valid regular expression, the latter is not).
5. E is the complement of a language that is recursively enumerable but not recursive
6. $NA_{TM} = \{ \langle M, w \rangle \mid M \text{ is a Turing machine and } w \notin L(M) \}$
7. $5CLIQUE = \{ \langle G \rangle \mid G \text{ has a complete subgraph on } k \geq 5 \text{ vertices} \}$
8. $DNF-SAT = \{ \langle F \rangle \mid F \text{ is a satisfiable Boolean function in the sum-of-products canonical form} \}$

Question 3 [15 marks].

1. Write the pumping lemma for context-free languages. Be careful in using the existential and universal quantifiers.
2. Write in point-form the basic ingredients of proving the above lemma.

Marks will be deducted for sloppy answers!

Question 4 [12 marks].

Consider the problem of testing whether the language recognized by an arbitrary given Turing machine M is finite. (a) Formulate this problem as a language, and (b) show that it is undecidable.

Question 5 [12 marks].

Consider the problem of testing whether a Boolean function in conjugative normal form has at least three satisfying assignments. (a) Formulate this problem as a language, and (b) show that it is NP-hard.

** The End **