CMPUT 474 Midterm
Feb. 15, 2001

70 Minutes for 60 Marks
You may use your text book.
You may not use any electronic device.

Answer all questions in the accompanying exam booklet. Hand in the exam question sheet with your exam.

No questions during the exam. If something is unclear, or appears to be an error, then clearly state your assumptions.

1. [marks 5] *Briefly* justify your answers to the following true or false questions about languages. All languages are over $\Sigma = \{0, 1\}$.

   (a) $\emptyset^* = \emptyset$

   (b) $(0^*1^*)^k = \{0, 1\}^*$, for $k \in \mathcal{N}^+$.

   (c) $0^* \bigcap 1^* = \emptyset$

   (d) $0^* \bigcup 1^* = \{0, 1\}^*$

   (e) $0^*(0^* \bigcup 1^*) = \{0, 1\}^*0^*$

2. [marks 3] Suppose L is a regular language with alphabet Σ. Give a high level description of a simple algorithm to tell whether $L = \Sigma^*$. Note, your algorithm must halt in finite time when given a finite description of a regular language L as input.

3. [marks 5] Write a regular expression over $\Sigma = \{a, b, c\}$ for the set of strings containing at least one a and at least one b.

4. [marks 6] Show that the language

   $$L = \{x | x \in \{0, 1\}^* \text{ and } x \text{ does not contain 0110 as a substring}\}$$

   is regular.

5. [marks 9] Let L be a language. Does $L^* = (L^*)^*$? Prove your answer.

6. [marks 15] Let $\Sigma = \{0, 1\}$. Design (draw) a DFA to accept the language L defined by

   $L = \{x | \#_0(x)$ is even **and** x represents a binary number divisible by $3\}$

   For this exercise, binary numbers may start with 0's as well as 1's. Hint: *to determine whether a number is divisible by 3, keep track of the congruency class modulo 3 as you read the number: if* $n \equiv 1 \bmod 3$ *what is* $2n$*?*

7. [marks 17] We say that a string x is a **prefix** of string y if a string z exists where $xz = y$ and that x is a **proper prefix** of y if in addition $x \neq y$. Prove that the class of regular languages is closed under the operation NOPREFIX defined below.

   $$\text{NOPREFIX}(L) = \{w | w \in L, \text{no proper prefix of } w \text{ is in } L\}$$

   Answer the following question first: if $\Lambda \in L$ what is NOPREFIX(L)?

Total Marks = 60.