CMPUT 474 Final Exam
Apr. 24, 2001

Time: 2 Hours
You may use your text book.
You may not use any electronic device.

Place all your answers in the accompanying exam booklet(s), and clearly identify which question you are answering.

I will answer NO QUESTIONS during the exam. If something is unclear, or appears to be an error, then clearly state your assumptions.

1. [marks 10] Is the following language regular? $L = \{x : x \in \{0,1\}^* \text{ and } (\#_0(x) \bmod 3) \neq (\#_1(x) \bmod 3)\}$. Justify your answer with an appropriate construction or proof.

2. [marks 10] Prove the following language is not regular using a Pumping Lemma. $L = \{x \diamond x^R : x \in \{0,1\}^*\}$.

3. [marks 8] Let $\Sigma = \{a, \ldots, z\}$ and consider the language L consisting of standard regular expressions over $\Sigma \bigcup \{(,), |, *\}$. Which of the following is the best characterization of L? Briefly justify your answer. (i) $L \in \mathcal{L}_{DFA}$ (ii) $L \in \mathcal{L}_{PDA}$ (iii) $L \in \mathcal{L}_{DTM}$. (iv) $L \in \mathcal{L}_{NTM}$.

4. [marks 7] Present a high level description of an NTM to accept the following language $L = \{0^i$: where i is a composite number $\}$.

5. [marks 10] True or False – Justify your answer. $\{a^n b^n a^m b^m : n, m \geq 0\}$ is a CFL?

6. [marks 15]

   - Formally define a *multi-headed Turing machine*, a model in which each work tape can have k tape heads.
   - Argue that a DTM with one work tape with one head can simulate a DTM with one work tape with two heads.

MORE Questions on the OTHER side.

7. [marks 10] Recall $G = G(V, E)$ is a graph and we use $n = |V|$ and $m = |E|$. Define HALF-CLIQUE $= \{G : G$ contains a clique of size $\geq n/2\}$. Show that HALF-CLIQUE is NP-complete. You may recall that Clique is NP-complete. You may choose to use either a logspace or a polytime reduction argument.

8. [marks 10]

   Consider the following puzzle. The input is a graph $G = (V, E)$, with $n = |V|$ and $m = |E|$. There is also a set of $2k$ markers placed on the vertices. k of the markers are red, and k are black, Each vertex may have any number of markers but they must all be the same color. That is, no vertex may have a red and black marker at the same time. A *move* in the puzzle consists of moving one marker of either color from a vertex to an adjacent vertex that either has no marker, or whose markers are of the same color as the marker being moved. Also as part of the input is an integer B. The object is to get all red markers together on one vertex, and all black markers on another using at most B moves.

   Define a suitable language for this puzzle, and show that it is in PSPACE.

Maximum Marks Available = 80.