c340 Exam 1

Feb 24, 2004

Name:

Student ID:

Instructions: Read through all questions first. Plan your time. (e.g. you might do all the conceptual steps first and leave calculations until the end.) Answer neatly and succinctly in the space provided. Show steps you wish to have graded, but not e.g. scribbles for numerical calculations etc. (If needed, use additional crib sheets or back sides to first work out your solution, then structure it and transfer it neatly to the exam). **Allowed:** 2 single sided sheets 8 1/2 by 11 inches with your own handwritten notes. These are to be handed in with your exam. Calculator.

1 Error propagation and analysis

1.1 General form of relative error propagation (2%)

For a function $y = f(x)$ give the formula (expressed in f) that given the relative error in x: $\frac{|\Delta x|}{|x|}$ computes the relative (forward) error in y: $\frac{|\Delta y|}{|y|}$.

1.2 Apply to lens focal length (3%)

The focal length of a lens is given by the lens formula: $\frac{1}{f} = \frac{1}{a} + \frac{1}{b}$
Derive an expression for the relative error in f, given those of a and b. Simplify your expression to as few terms as possible.

1.3 Particular lens (2%)

Given $a = 32mm \pm 3\%$ and $b = 46mm \pm 3\%$ compute f with error bound.

2 Linear equations (4%)

A well known supporter of Alberta beef is setting up a restaurant. To not lose out on the mad cow scare however, he hedges by also offering some BC fish. A beef meal takes 1/6 hour to prepare and the ingredients cost $ 0.89. A fish meal takes 1/5 hour to prepare and costs $ 1.11. Assuming the restaurant is open for 11 hours and the daily ingredient budget is $ 60, how many beef and fish meals can be prepared if the resources are utilized fully?

3 Data fitting

In a particular data fitting problem the parameters a and b are to be determined for a function $f(x) = \frac{a}{1+bx}$ and given data:

x_i	0	1	3	5
f_i	1	0.5	0.2	0.1

3.1 Problem transformation (2%)

Linearize this problem and find a suitable variable transform.

3.2 Parameter fitting (5%)

Formulate and solve the overdetermined equation system using the linearized model and the data above. Then compute a and b. Does your solution minimize $\|f_i - \frac{a}{1+bx_i}\|_2$ for the given data points?

4 Eigenvalue problems

A recursion over a_k and b_k, where $a_0 = 3$ and $b_0 = 1$ is defined as

$$a_{k+1} = a_k + 2b_k$$

$$b_{k+1} = 2a_k + b_k$$

4.1 Matrix form (2%)

Formulate the above recursion in matrix form for a matrix M and vector $x_k = [a_k, b_k]^T$.

4.2 Power method (6%)

a. Calculate x_4 and x_5 through direct application of M on $x_0 = [a_0, b_0]^T$.

b. What are approximate values of the largest eigenvalue $\lambda_{\max}$ and eigen vector $v_{\max}$ for M?

c. How can you now approximate x_{20} without performing matrix multiplications with M? What is the approximate x_{20}?