c340 Exam 2

Apr 6, 2004

Name:

Student ID:

Instructions: Read through all questions first. Plan your time. (e.g. you might do all the conceptual steps first and leave calculations until the end.) Answer neatly and succinctly in the space provided. Show steps you wish to have graded, but not e.g. scribbles for numerical calculations etc. (If needed, use additional crib sheets or back sides to first work out your solution, then structure it and transfer it neatly to the exam). **Allowed:** 4 single sided letter size sheets with your own notes. These are to be handed in with your exam. Non-programmable numeric-only calculator.

1 Derivatives and Richardson extrapolation (6%)

An automobile accelerates from a stop while the passenger registers the speed every second as follows:

t	0	1	2	3	4	5	6	7	8	9
v	0	3.61	7.22	10.10	12.50	14.62	16.60	18.06	19.54	20.28

Use the centered difference formula and two steps of Richardson extrapolation to accurately compute the acceleration $\frac{dv}{dt}$ at $t = 4$. (From the acceleration and the weight one can e.g. compute the engine power.)

2 Non-linear least squares (10%)

Recent events have shown a need to train politicians in nuclear threat evaluation. In a lab demonstration a substance containing equal parts of two radioactive compounds has been mixed. The composite radioactive intensity decays according to the function:

$$I(t) = e^{\lambda_1 t} + e^{\lambda_2 t}$$

Show George and Tony how to identify the radioactive decay constants λ_1 and λ_2.
a. For the data fitting problem assume n pairs of data (t_k, I_k) has been measured. Formulate the residual to be minimized and Jacobian (using the data points t_k and I_k).

b. Using a Geiger counter the following radioactivity was measured:

t_k	1	2	3	4
I_k	1.15	0.74	0.52	0.38

Calculate one step in Gauss-Newton's method using a starting value of $[0.4, 1]^T$ for the lambdas.

c. Why is $[1, 1]^T$ not a good starting value for the lambdas?

3 Differential equations (10%)

Given the non-linear BVP: $y(0) = 0.5$, $y(1) = 0$

$$-y''(x) + \cos^2(y(x)) = 4x$$

a. Given a discretization of the interval into n+1 segments $0, 1, \ldots, n+1$, (with n interior points $y_1, \ldots, y_k, \ldots, y_n$) show the finite difference equation for the BVP.

b. Show that for $h = 0.5$ the equation for the interior point is of the form $8y + \cos^2(y) - C = 0$, for some constant C (i.e. find C). Solve for y using Newton's method with a starting value of 1 to four correct decimals. How many iterations do you need?