Instructor: Lorna Stewart

CMPUT 304 Fall 2000 Midterm Exam

Date: Tues, Oct 24
Time allowed: 75 minutes
Total Marks: [50]

Question 1: [10] Show how the strongly connected components algorithm on page 489 of the textbook works on the graph below. Specifically, show the forest and the finishing times computed in line 1 and the forest produced in line 3. Assume that the loop of lines 5-7 of DFS considers vertices in alphabetical order and that the adjacency lists are in alphabetical order.

Question 2: [10] Suppose $A_n = \{a_1, a_2, \ldots, a_n\}$ is a set of distinct coin types, where each a_i is a positive integer, for $1 \leq i \leq n$. Suppose also that $a_1 < a_2 < \ldots < a_n$ and that $a_1 = 1$. The coin-changing problem is defined as follows. Given a natural number C, find the smallest number of coins from A_n that add up to C, given that an unlimited number of coins of each type are available.

Design a dynamic programming algorithm that, on inputs A_n and C, outputs the minimum *number* of coins needed. Clearly describe the data structure used for storing results, the initialization, the location of the final answer, the order in which the partial solutions are computed, exact formulas for the computation, and the asymptotic complexity of your algorithm.

Question 3: [10] Give an upper bound on the number of times the **while** loop of lines 6-7 of Algorithm KMP-Matcher (page 871 of the textbook) will execute, as a function of m. Give an example of P and T that demonstrates that your bound is tight.

Question 4: [10] In a large university with k academic departments, we must appoint an important committee. One professor will be chosen from each department. Some professors have joint appointments in two or more departments, but each must be the designated representative of at most one department. Every professor is an assistant professor, associate professor, or full professor. We must use equally many assistant professors, associate professors, and full professors among the chosen representatives (assume that k is divisible by 3). Express the problem of choosing the committee as a network flow problem, in which units of flow correspond to professors chosen for the committee.

Clearly identify the flow network, the source and sink, the edge capacities, and how to use the network to test whether such a committee exists and find it if it does.

Question 5: [10] Define the optimization problem LONGEST-PATH-LENGTH as the relation that associates each instance of an undirected graph and two vertices with the length of a longest simple path in the graph between the two vertices. Define the decision problem

LONGEST-PATH:
Instance: Graph $G = (V, E)$, $u, v \in V$, and integer k.
Question: Is there a simple path from u to v in G whose length is at least k?

(a) [6] Show that the optimization problem LONGEST-PATH-LENGTH can be solved in polynomial time if and only if LONGEST-PATH $\in$ P.

(b) [4] Show that LONGEST-PATH $\in$ NP.

(c) [0] Is LONGEST-PATH $\in$ P? (Give me your best guess.)

YES NO