CMPUT 272 Winter 2001: Section B3
Midterm
Thursday, March 8
Time: 50 minutes
Weight: 20%
Total Points: 35

Last name:
First name:
Unix ID:

Instructions:

- This exam is **open book**
- No calculators or other mechanical devices are allowed.
- This quiz should have 6 pages and 4 questions. You are responsible for checking that your exam booklet is complete.
- Either HR or GT notation may be used in the derivations.
- Note that for many questions **much more space** has been provided than is needed. Do not feel obligated to make your answer fill the space.

Question 1 [4 points] For each of the following formulas indicate whether it is a tautology, contradiction, or contingency. No justification is required.

1. $\forall x\ (P(x) \Rightarrow Q(x)) \wedge \neg\forall x\ (P(x) \Rightarrow Q(x))$
2. $\forall x\ (P(x) \Rightarrow Q(x)) \vee \neg\forall x\ (P(x) \Rightarrow Q(x))$
3. $(A \vee C \Rightarrow B) \Rightarrow (A \Rightarrow B)$
4. $(\neg A \wedge \neg B) \Rightarrow A$

Question 2 [12 points]

Consider the following program which does integer division. Its inputs are the natural numbers X and *div*, and its outputs are the quotient *quot* and the remainder *rem* from the division of X by *div*. E.g. inputs of $X = 13$ $div = 3$ result in $quot = 4$ and $rem = 1$ since $13 = 3 \cdot 4 + 1$.

Integer Division

Int X, div, quot rem;
Preconditions: $X \geq 0$ & $div > 0$
$rem, quot := X, 0;$
do
  Variant: *rem*
  Invariant: ?
  [] $rem \geq div \longrightarrow rem, quot := rem - div, quot + 1;$
  [] $rem < div \overset{exit}{\longrightarrow}$
od
Postconditions: $X = div \cdot quot + rem$ & $0 \leq rem < div$

2.a [2 pts]: Trace the execution of this algorithm for the inputs $X = 13$ and $div = 3$. Show the intermediate values of *rem* and *quot* at the beginning of each loop.

2.b [3 pts]: State the invariant for the loop.

2.c [7 pts]: Prove that the variant for the loop is always nonnegative

Question 3 [13 points]
Consider the two formulas:

π: `(ex x st Smiles[x]) implies (for y holds Happy[y])`
ψ: `for z holds Smiles[z] implies Happy[z]`

3.a [9 pts]: Provide a derivation showing that π logically implies ψ. Only basic rules are permitted. Annotate each step.

```
environ
  reserve x, y, z for PERSON;
  pi: (ex x st Smiles[x]) implies (for y holds Happy[y]);

begin
```

3.b [4 pts]: Provide an interpretation showing that ψ does not logically imply π.

Question 4 [6 points] Provide a derivation showing that `(P[] & Q[]) implies R[]` logically implies `P[] implies (Q[] implies R[])`. Only basic rules are permitted. Annotate each step.

```
environ
  premise: (P[] & Q[]) implies R[];
begin
```

Qu	Mark	
1		4
2		12
3		13
4		6
Σ		35