COMPUTING SCIENCE 272

SECTION B3

MIDTERM EXAMINATION II

DATE: March 14, 2000 TIME: 80 MINUTES

INSTRUCTOR: I. E. LEONARD

Question One [3+3]

The universe of discourse for this problem is the set of all integers

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}.$$

(a) Give an example of a binary predicate $P(x, y)$ for which the quantified statement

$$\forall\, x\, \exists\, y\, P(x, y)$$

is true, while the quantified statement

$$\exists\, y\, \forall\, x\, P(x, y)$$

is false.

(b) Can you find a binary predicate $P(x, y)$ for which the quantified statement

$$\exists\, y\, \forall\, x\, P(x, y)$$

is true, while the quantified statement

$$\forall\, x\, \exists\, y\, P(x, y)$$

is false? If so, give an example; if not, give a proof.

Question Two [2+2+2+2]

In this problem, the universe of discourse is the set of positive integers

$$\mathbb{N} = \{1, 2, 3, \ldots\}.$$

Determine whether the following are true or false. If true, give a proof; if false, give a counterexample.

(a) $\forall\, m\, \exists\, n\, (m < n)$.

(b) $\exists\, n\, \forall\, m\, (m < n)$.

(c) $\forall\, n\, \exists\, m\, (m < n)$.

(d) $\exists\, m\, \forall\, n\, (m < n)$.

Question Three [6]

Show that set intersection distributes over symmetric difference, that is,

$$A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$$

for any sets A, B, C.

Question Four [3+3]

Let $A_1, A_2, \ldots, A_n$ be a finite collection of subsets of a universal set $\mathcal{U}$, prove DeMorgan's laws

(a) $\left(\bigcup_{k=1}^{n} A_k\right)^c = \bigcap_{k=1}^{n} A_k^c.$

(b) $\left(\bigcap_{k=1}^{n} A_k\right)^c = \bigcup_{k=1}^{n} A_k^c.$

Question Five [4]

Von Neumann's definition of an ordered pair (a, b) for elements a and b from a universal set $\mathcal{U}$ is as follows:

$$(a, b) = \{\{a\}, \{a, b\}\}.$$

Use this definition to show that if $(a, b) = (a', b')$, then $a = a'$ and $b = b'$.