COMPUTING SCIENCE 272

SECTION B3

MIDTERM EXAMINATION I

DATE: FEBRUARY 8, 2000 TIME: 50 MINUTES

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Question One [2+2+2]

Nand is a logical connective defined by the truth table below.

p	q	p *nand* q
0	0	1
0	1	1
1	0	1
1	1	0

That is, p *nand* q means "not (p and q)." In this problem you are to show that all compound propositions can be written using only this connective.

(a) Find a proposition logically equivalent to $\neg p$ using only the connective *nand*.

(b) Find a proposition logically equivalent to $p \wedge q$ using only the connective *nand*.

(c) Find a proposition logically equivalent to $p \vee q$ using only the connective *nand*.

Question Two [6]

In a new case, Sherlock Holmes has discovered that

(i) The cook or the butler was in the kitchen.

(ii) The cook was in the kitchen or in the dining room.

(iii) If the butler was smoking a cigar, he was not in the kitchen.

(iv) If the cook was not in the dining room, the butler was not smoking a cigar.

Define the following propositions:

$b =$ "The butler was in the kitchen."

$c =$ "The cook was in the kitchen."

$d =$ "The cook was in the dining room."

$s =$ "The butler was smoking a cigar."

Which of the propositions b, c, d, s, or their negations, can Holmes deduce from these facts? You must list the reasons, that is, the rules of inference, for each of the statements in your proof.

Question Three [2+2+2+2]

Using the following predicates on the universal set of all persons,

Predicate	**Intended Meaning**
Mother(A, B)	A is the mother of B
Father(A, B)	A is the father of B
Female(A)	A is female
Male(A)	A is male
Sister(A, B)	A is the sister of B
Brother(A, B)	A is the brother of B
Older(A, B)	A is older than B

express each of the following propositions in symbolic form:

(a) No person is older than that person's father.
(b) Some persons have no younger brothers.
(c) If any person A has the same mother and father as another person B, and if A is female, then A is a sister of B.
(d) If any person A is the brother or sister of another person B, then A has the same mother as B, or A has the same father as B.

Question Four [3+3]

(a) The universe of discourse for this problem is the set of all integers

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}.$$

Give an example of a binary predicate $P(x, y)$ for which the quantified statement

$$\forall\, x\, \exists\, y\; P(x, y)$$

is true, while the quantified statement

$$\exists\, y\, \forall\, x\; P(x, y)$$

is false.

(b) The universe of discourse for this problem is the set $\mathbb{R}$ of all real numbers.
In the arithmetic of real numbers, there is a real number, namely 0, called the *additive identity* since $a + 0 = 0 + a = a$ for every real number a. This may be expressed symbolically as

$$\exists\, z\, \forall\, a\; \big(a + z = z + a = a\big).$$

In conjunction with the existence of an additive identity is the existence of *additive inverses.* Write a quantified statement that expresses

"Every real number has an additive inverse."

Do not use the minus sign anywhere in your statement.

Question Five [4]

In mathematics, one often wants to assert not only the existence of an object a (be it a number, a triangle, or whatever) that satisfies a statement $P(x)$, but also the fact that this object a is the only such object for which $P(x)$ is true. Hence the object is *unique.* This is denoted by the quantifier

$$\exists\, !\, x$$

which is read "There exists a unique x." Show that this quantifier can be written using only the existential and universal quantifiers, that is, find an expression containing $\exists$ and $\forall$ that is logically equivalent to $\exists\, !\, x\; P(x)$.