CMPUT 272 Midterm [B2 -- Harms]

March 8, 2001

NAME: _____ UserId: _____

Comments:

- This exam is worth 20% of your final grade. There are 4 questions and 5 pages. The mark distribution is given beside the questions. The total number of marks is 40.

- This is a closed book exam. You are allowed one sheet of paper (size 8 1/2 x 11 in) containing information of your choice. No calculators are allowed.

- This must be your individual work.

- Show your work! Good Luck!

Question 1 [10 marks]

This is a question on translations. The universe of discourse for this question is a set of inhabitants of the Island of Knights and Knaves. Let the predicate **Knight[x]** mean that Islander **x** is a knight. Some possibilities for the number of knights are:

a nobody
b at most one
c exactly one
d at least one
e at most two
f exactly two
g at least two
h everyone

Which of the above possibilities a-h is **best** described by each of the following formulas?

environ
reserve x, y for Islander;
given Lancelot, Galahad being Islander;

0: Lancelot < > Galahad;

== Best describes
== possibility:

1: Knight[Lancelot] & (for x holds Knight[x] implies x=Lancelot) == ____

2: Knight[Lancelot] & (for x holds Knight[x] or x=Lancelot) == ____

3: ex x st Knight[x] == ____

4: for x holds (Knight[x] iff (x=Lancelot or x=Galahad)) == ____

5: for x holds (Knight[x] implies x=Lancelot) == ____

6: Knight[Lancelot] & not (for x holds (Knight[x] implies x=Lancelot)) == ____

7: not (ex x st Knight[x]) == ____

8: for x holds (ex y st x<>y & Knight[y]) == ____

9: for x holds (Knight[x] implies (x=Lancelot or x=Galahad)) == ____

10: (for x holds Knight[x] implies x=Lancelot) implies Knight[Lancelot] == ____

Question 2 [10 marks]

Consider the following program:

Nat x, y;
Preconditions: None
Nat i;

i, y := 0, 0;
do
Variant: ?
Invariant: ?

[] $i < x \rightarrow y, i := y + 2 \cdot i + 1, i + 1$

[] $i = x \rightarrow$ exit
od

Postcondition: $y = x^2$

Part a [2 points]: Find a variant for the loop.

Part b [2 points]: Find the invariant for the loop that verifies the postcondition when the loop exits.

Part c [2 points]: (Base Case) Show that the invariant is true upon first entering the loop

Part d [4 points]: Show that if the invariant is true at an arbitrary time through the loop and if the loop does not exit the next time through the loop then the invariant will stay true.

Question 3 [10 marks]

Complete the following proof. Include one step for each rule of inference applied and mark in a comment what rule was used. Do not leave out any steps. You can use derived rules as well as basic rules.

environ
    reserve x, y, z for Person;

A: for x holds (Rich[x] implies not Happy[x]);

begin == prove that (ex y st Rich[y]) implies not (for z holds Happy[z])

Question 4 [10 marks]

Complete the following proof. Do not use more than 1 rule per inference step. Mark in a comment what rule was used. You can use derived rules (as well as basic rules).

environ
reserve x,y for Person;

A1: for x holds (Rich [x] implies Happy [x]);

A2: for y holds (Funny [y] implies Laughs [y]);

begin == prove that for x holds ((Rich[x] or Funny[x]) implies (Happy[x] or Laughs[x]))