CMPUT 272, Section B2, Second Midterm Examination W.W. Armstrong
March 14, 2000

Your name: ______________________________ SID: ______________________

Instructions: Write your answers on the question sheets. Do not communicate to anyone in the exam room at any time, even while leaving. Don't ask the instructor any questions about the exam until all exams are handed in. If you don't understand a question, make an assumption about what was intended. If you prove convincingly there is a mistake in a question, you can also get full marks for it! There are several slightly different question sheets being used, so **COPYING IS CHEATING AND VERY RISKY.** The exam is open books, open notes.

Total: 30 points. This is worth 15% of the mark in the course. **Time:** 45 minutes.

Question 1 (10 points)
Complete the following proof in Mizar notation. Minor syntax problems will be ignored.

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environ
          reserve A, B, C, x for THING;
EqualDef:       for A, B holds Equal[A, B] iff
                (for x holds In[x, A] iff In[x, B]);
UnionDef:     for A, B, C holds Union[A, B, C] iff
              (for x holds In[x, A] or In[x, B] iff In[x,C]);
begin
Problem: for A, B st Equal[A, B] holds Union[A, A, B]
proof
s1: now let M, N be THING;
 1: Union[M, M, N]
   iff (for x holds In[x,M] or In[x,M] iff In[x,N]) by UnionDef;
2: Equal[M,N] iff (for x holds In[x,M] iff In[x,N]) by EqualDef;
s3: now assume 3: Equal[M,N];
        4: (for x holds In[x,M] iff In[x,N] ) by 2,3;
        s5: now let y be THING;
                5: In[y,M] iff In[y,N] by 4;
                thus In[y,M] or In[y,M] iff In[y,N] by 5;
        end;

        6: ______________________________________________ by s5;

        thus Union[M,M,N] by 1,6;

        end;

thus ____________________________________________________

end;                                                     by s3;

thus ______________________________________________________
end
                                                         by s1;
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CONTINUED ON OTHER SIDE....

Question 2 (6 points) Make two Venn Diagrams of three sets A, B, C in general position inside a universe of discourse E. Darken the following areas of the diagrams 1 and 2.

1. $(A \cap B \cap C) \cap (A^c \cup B^c \cup C^c)$ where the c-superscript means complementation in E.

2. $((A - B) - C) \cup ((B - C) - A)$

Question 3 (4 points) If P(x), Q(x), and R(x) are the indicator functions (or characteristic conditions in GT) of three sets A, B, C respectively, what is the indicator function of $(A \cup B) \cap (E - C)$, where E is the universe of discourse? You can use operations of set theory in defining the indicator function.

Question 4 (6 points)

a. What exactly is the equivalence class of 16 modulo 7. Use set-builder notation. Let Rem(a,b) denote the smallest non-negative remainder when you divide a by b.

b. In arithmetic modulo 5, what is [3] + [2] + [4] ? Ans: ______________________ (smallest non-negative representative please)

c. In arithmetic modulo 5, what is [3] * [2] * [4] ? Ans: ______________________(smallest non-negative representative please)

Question 5 (4 points) Use a Venn Diagram to solve the following puzzle:

A history class with 19 people in it have some who are artists, some who are builders, and some who are contractors. These three groups don't overlap except for two people who are artists, builders, and contractors. There are three more artists than there are builders, and there is one fewer contractor than there are builders. How many people are in the class who are artists? Builders? Contractors?