

**CMPUT 272, Section B2, First Midterm Examination** *W.W. Armstrong*  
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**Instructions:** Write your answers in the booklet provided, not on the question sheets. Hand in the answer booklet and keep the question sheet when you are finished the exam. Do not communicate to anyone in the exam room at any time, even while leaving. Don't ask the instructor any questions about the exam until all answer booklets are handed in. If you don't understand a question, make an assumption about what was intended. If you prove convincingly there is a mistake in a question, you can also get full marks for it! There are several slightly different question sheets being used, so **COPYING IS CHEATING AND VERY RISKY**. The exam is open books, open notes.

**Total:** 30 points. This is worth 15% of the mark in the course. **Time:** 45 minutes.

**Question 1** (10 points): Write in your answer booklet the line numbers justifying each line as required. For a correct justification, all line numbers needed must be given, and no extra ones, to get one point. Format:  
 2: by a4,1;  
 3: by ... etc.

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environ
reserve x for PERSON;
given Alice, Bob being PERSON;
a1: for x st L[x] or C[x] or S[x] holds not E[x];
a2: not E[Bob] implies S[Alice];
a3: for x st CS[x] holds E[x];
a4: not CS[Alice] or C[Bob];
a5: for x holds N[x] iff not CS[x];
begin
s1: now
    assume 1: CS[Alice];
    2: C[Bob] by a4,1;
    3: not E[Bob] by ;
    4: S[Alice] by ;
    5: not E[Alice] by ;
    6: CS[Alice] implies E[Alice] by ;
    7: not CS[Alice] by ;
    8: CS[Alice] by ;
    thus contradiction by ;
end;
9: not CS[Alice] by ;
10: N[Alice] iff not CS[Alice] by ;
11: N[Alice] by ;
    
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Possible meanings of the predicates (optional):  
 L[x] means "x lies",  
 C[x] means "x cheats"  
 S[x] means "x steals"  
 E[x] means "x is ethical"  
 CS[x] means "x is a computer scientist"  
 N[x] means "x is normal"

**Important:** remember that the precedence rules are different in the GT text and in MIZAR.



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**Question 2** (5 points): Consider the domain of positive natural numbers: 1,2,3, ... and so on. Let P(x) mean x is prime, and let E(x) mean x is even. Let LT(x,y) mean "x is less than y". Translate into predicate logic the following statements:

- a.) A prime number exists.
- b.) The number 1 is less than all even numbers (in the domain, of course).
- c.) All even prime numbers are less than 3.
- d.) There is no prime number less than 2.
- e.) All prime numbers which are not less than 3 are not even.

**Question 3** (5 points): Copy the following formula into your answer booklet and circle all the free variable occurrences in it (the ones you don't circle will be considered bound):

$$\forall x(\exists y (P(x,z) \wedge Q(y,z)) \vee R(y))$$

CONTINUED ON OTHER SIDE....

**Question 4** (5 points) Make a truth table to determine whether or not the following is a sound argument:

$$P \vee Q, P \Rightarrow R, Q \Rightarrow P \vdash R$$

Explain what property it is of your truth table that shows the argument is sound.

**Question 5** (5 points) Consider the substitution

$$S_x^y ((\forall x P(x)) \Rightarrow Q(x,y)).$$

You must be careful in doing this to respect the bindings of the variables. (The textbook may not be perfectly clear on this point, but the lectures were!)

Which of the following is a formula that is logically equivalent to the result of substitution? (Write it down in your answer booklet).

1.  $((\forall x P(x)) \Rightarrow Q(x,y))$
2.  $((\forall x P(x)) \Rightarrow Q(y,y))$
3.  $((\forall y P(y)) \Rightarrow Q(x,y))$
4.  $((\forall y P(x)) \Rightarrow Q(y,y))$
5. none of the above