Department of Computing Science
University of Alberta
CMPUT 272 Final Exam for Section B2
Instructor: W. W. Armstrong
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INSTRUCTIONS: Please write all the answers in the answer booklet provided, not on this sheet. You may keep this sheet after the exam. Do not talk in the examination hall, even when you are leaving. MIZAR notation is not required, but may be used in any of the questions. **Don't ask any questions about the exam** – it has been carefully checked. If you don't understand something, you can make an assumption, state it, and continue your answer. The total is 100 points. **Several different question sheets are being used.**

WHAT IS ALLOWED: All documentation is allowed, books, assignment solutions, notes etc. No means of communication may be used.

Question 1 [10 points] Make one or more Venn Diagrams, each with sets A, B, C in general position, to show how to express A\(B∪C), the set difference of A and (B∪C), in terms of A\B and A\C. Give the equation that shows how to express A\(B∪C) this way.

Question 2 [10 points] Does implication distribute over disjunction from the right (left)? Answer for both left and right, with truth tables or some other form of proof.

Question 3 [10 points]

a.) They say "All the world loves a lover". We interpret a lover as someone x such that there exists someone y satisfying L[x,y], meaning "x loves y". There is some ambiguity in "All the world loves a lover". Give a translation of this sentence into a statement of predicate logic which expresses the idea that anyone x who loves anyone y (y being x or anyone else) is loved by everyone.

b.) Apply this to a universe of discourse containing just three persons: John, George, and Alice. Give an interpretation satisfying "All the world loves a lover".

c.) How many more interpretations are there than the one you gave above? If any exist, describe them.

Question 4 [10 points]. Prove by induction that for all natural numbers n

$5^{n+1} = 1 + 4\,(5^0 + \ldots + 5^n)$.

Question 5 [10 points]. We use $\lceil x \rceil$ to denote the ceiling of x, for all real numbers x.
a.) Write out the first four terms of the recursively defined sequence given by $T(0) = 0$ and $T(n) = T(\lceil (n-1)/3 \rceil) + 1$ for $n > 0$.

b.) Prove that for this sequence, $T(n) \leq n$ for all natural numbers n. Hint: $\lceil m/3 \rceil$ = one of the numbers m/3, (m+1)/3, (m+2)/3, whichever is an integer.

c.) What is this proof technique called?

Question 6 [10 points] Consider arithmetic modulo 7. Let the equivalence class of m modulo 7 be denoted [m]. If possible, give the answers to the following arithmetic questions in the format [n], where n is a representative of the class between 0 and 6:

a.) [6] + [6] = ?
b.) [12] * [3] = ?
c.) [12] / [5] = ?
d.) [1] / [7] = ?
e.) If $[n]^2 = [-3]$, what is [n]? Give two answers, if possible

Question 7 [10 points] Suppose the cardinality of A is 5. Suppose f is a function from A to A, and f ° f is f composed with itself. We assume the range of f is a subset of the domain of f.

a.) If f is not surjective, can f ° f be surjective? Explain.
b.) If f is not injective, can f ° f be injective? Explain.
c.) If f is not total, can f ° f be total? Explain.
d.) If f is injective must f ° f be injective? Explain.
e.) If f is surjective must f ° f be bijective? Explain.

Question 8 [10 points] A relation is generally a subset of a Cartesian product. This concept can be used to store data in a relational database. Now suppose there are 15 tuples in a relation on attributes A, B, C where A has 5 possible values, B has 3 and C has 2. What fraction of the Cartesian product's tuples are in the relation?

Question 9 [10 points] A puzzle: I am thinking of an integer *n* such that if I multiply it by the integer which is one less, I get 42. The integer I am thinking of is divisible by 3. What integer *n* am I thinking of? Or does this question not have an answer?

Question 10 [10 points] Suppose R is a relation on A, where A is any finite set. Suppose R is both symmetric and antisymmetric.

a.) What can you say about the general form of R?
b.) How many such relations R are there as a function of the cardinality of A?

To all students of section B2: Please check your marks on ugweb.cs.ualberta.ca/~c272 or www.cs.ualberta.ca/~arms/c272 to make sure everything is ready for calculating the final grades. I wish you a nice summer, and lots of success in the future! WWA