CMPUT 272 Winter 2001: Section B1
Midterm
Wednesday, March 7
Time: 50 minutes
Weight: 20%
Total Points: 35

Last name:
First name:
Unix ID:

Instructions:

- This exam is **open book**
- No calculators or other mechanical devices are allowed.
- This quiz should have 5 pages and 5 questions. You are responsible for checking that your exam booklet is complete.
- Either HR or GT notation may be used in the derivations.

Question 1 [4 points] Which of the following formulas are logically equivalent? (There may be more than one equivalent pair)

1. $\neg\forall x(\exists x\ [Q(x) \vee R(x)] \Rightarrow \forall x\ P(x))$

2. $\neg\forall x(\exists y\ [Q(x) \vee R(y)] \Rightarrow \forall x\ P(y))$

3. $\neg\forall x(\exists y\ [Q(x) \vee R(y)] \Rightarrow \forall y\ P(x))$

4. $\neg\forall x(\exists y\ [R(y) \vee Q(y)] \Rightarrow \forall x\ P(x))$

5. $\neg\forall y(\exists x\ [R(y) \vee Q(x)] \Rightarrow \forall x\ P(y))$

Question 2 [4 points] Consider the following incorrect proof:

```
environ
  reserve x for PERSON;

begin
  a0: now
    assume b0: ex x st Happy[x];
    consider Bob being PERSON such that
    b1: Happy[Bob]                by b0;
    b2: now
      let Bob be PERSON;
      thus Happy[Bob]             by b1;
    end;
    thus for x holds Happy[x]     by b2;
  end;
  conclusion: (ex x st Happy[x]) implies (for x holds Happy[x])   by a0;
```

2.a [2 pts]: Briefly state what is wrong with this "proof".

2.b [2 pts]: Find an interpretation that demonstrates the conclusion is not a tautology.

Question 3 [8 points]

3.a [6 pts]: Provide a derivation showing that `P[] implies (R[] implies Q[])` logically implies `R[] implies (P[] implies Q[])`. Only basic rules are permitted. Annotate each step.

```
environ
  prem: P[] implies (R[] implies Q[]);

begin
```

3.b [2 pts]: Are the two formulas in part **a.** logically equivalent? Briefly state why or why not.

Question 4 [8 points]

Consider the following program which converts a number into binary form. Its input is the natural number X, and its output is an array of ones and zeros $A[]$ holding the binary representation of X.

div is the integer division function. rem is the function which returns the remainder of an integer division. e.g. since $13 = 2 \cdot 6 + 1$ we have that $13 \text{ div } 2 = 6$ and $13 \text{ rem } 2 = 1$.

Conversion to Binary

Nat X, n, A[0..n-1];
Preconditions: $X < 2^n$
Nat x, y;
$x, y := X, 0;$
do
    Variant: ?
    Invariant: $\sum_{i=0}^{y-1} A[i] \cdot 2^i + x \cdot 2^y = X$
  $[]\ y < n \text{ or } x > 0 \longrightarrow A[y], x, y := x \text{ rem } 2, x \text{ div } 2, y + 1;$
  $[]\ y = n \ \&\ x = 0 \xrightarrow{exit}$
od
Postconditions: ?

4.a [3 pts]: Trace the execution of this algorithm for the inputs $X = 13$, $n = 5$. Show the intermediate values of x, y and $A[]$ at the beginning of each loop.

4.b [2 pts]: State two possible variants for the loop.

4.c [2 pts]: State the postconditions for the loop. It must not involve any local variables.

4.d [1 pt]: What does the expression $\sum_{i=0}^{y-1} A[i] \cdot 2^i$ represent?

Question 5 [11 points]

5.a [9 pts]: Provide a derivation showing that `ex x st (for y holds P[x,y])` logically implies `for w holds (ex z st P[z,w])`. Only basic rules are permitted. Annotate each step.

```
environ
  reserve w, x, y, z for THING;
  prem: ex x st (for y holds P[x,y]);

begin
```

5.b [2 pts]: Are the two formulas in part **a.** logically equivalent? Briefly state why or why not.

Qu	Mark	
1		4
2		4
3		8
4		8
5		11
Σ		35