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## **COMPUTING SCIENCE 272 (B1)**

## FINAL EXAM (Winter 2000)

Instructor: John Willson

I.D.#	TIME: 120 Minutes	
LAST NAME:	FIRST NAME:	_
SIGNATURE:		PAGE
INSTRUCTIONS:	TOTAL MARKS: 100 Marks	ES: 11

- Please do not open exam until instructed to do so.
- No calculators are allowed; however, notes, and the text are allowed.
- No discussions during the exam are allowed. This is individual work only.
- Show all work for partial marks.
- Make sure there are 10 questions.
- If you require more work paper than the back of the pages, then ask for an examination book
- If there are multiple answers to a question do one answer and mention the other.

1	10
3	10
3	10
4	10
5	10
6	10
7	10
8	10
9	10
10	10
	100

1. (10) Using logical reasoning, prove or disprove the following "Of all the rectangles with a given perimeter, the square has the greatest area". (Hint: You do not have to put this question in terms of propositional, predicate, and/or set theoretic logic)

2. (10) Consider the connective  $\downarrow$  called the joint denial with  $p \downarrow q$  being read as "Neither p nor q". Construct the truth table for this connective and prove that the standard three connectives:  $\neg$ ,  $\wedge$ , and  $\vee$  can be expressed in terms of the  $\downarrow$  connective. Finally prove the contra-positive of  $p \rightarrow q$  using the  $\downarrow$ 

3. (10) Use the laws of Table 2.4 and propositional logic to derive

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$$\exists x P(x) \land \forall x \forall y (P(x) \land P(y)) \Rightarrow (x = y)$$

from

$$\exists x (P(x) \land \forall y (P(y) \Rightarrow (x = y)))$$

4. (10) Symbolize, then justify using Predicate Logic, the following reasoning. "None of the primes are integrally divisible by an even integer greater than 2; any of the primes is integrally divisible by the number 1; there exist some primes; therefore, 1 is not integrally divisible by an even integer greater than 2"

5. (10) Consider the following three sets:  $A = \{1,2,3\}$ ,  $B = \{3,4,5\}$ , and  $C = \{1,3,5\}$  Using a series of Venn diagrams and these three sets, pictorially verify the distributive law

 $A \cup (B \cap C) \equiv (A \cup B) \cap (A \cup C)$ 

6. (10) Proof the following:

"If two sets A and B, where  $A \cap B = \emptyset$ , are countable, then so is  $C = A \cup B$ "

7. (10) Two equivalence relations R and S are given by their relation matrices  $M_R$  and  $M_S$ . Show that R o S is not an equivalence relation.

$$M_{R} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{M}_{s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

8. (10) List all possible functions from  $X = \{a, b, c\}$  to  $Y = \{0, 1, 2\}$  and indicate in each case whether the function is one-to-one, onto, or one-to-one onto.

9.(10) Consider the following series

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{n}{(n+1)!}$$

Whose value is 1/2, 5/6, 23/24 for n = 1, 2, 3 respectively. Guess the general law (by observing more values if necessary) and prove your guess using mathematical induction.

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10. (10) Using SQL express the following relational operators of relational algebra:

a) Product P x Q (Also called the Cartesian Product) is:

b) Difference P - Q is: