

CMPUT 204 Section B3 (Basu)
MIDTERM Feb. 12, 2001
CLOSED BOOK. NO Notes or Calculators.
Time 50 minutes.
Answer all questions in the space provided.
Do scratch work on page backs.

Last Name: _____

First Name: _____

Make sure your name and ID is on the top of each internal page

Question 1 *Marks 10*

Which of the following are true? Indicate briefly why or why not. (Assume $f(n)$ and $g(n)$ are positive non-decreasing functions.)

1.a [2 marks] if $f(n) \in O(n^2)$ and $g(n) \in O(n^2)$ then $\frac{f(n)}{g(n)} \in O(1)$.

1.b [2 marks] $n^\alpha \in \Theta(n \log n)$ for some $\alpha > 1$.

1.c [2 marks] if $f(n) \in O(g(n))$ then $f(n) + g(n) \in \Theta(g(n))$.

1.d [2 marks] if $f(n) \in O(n)$ and $g(n) \in \Theta(n)$ then $2^{f(n)} \in O(2^{g(n)})$.

1.e [2 marks] $\log(\sqrt{n}) \in o(\log n^2)$.

Question 2 Marks 18

Consider sorting a list of n numbers using Quicksort.

2.a [6 marks] Let $A(n)$ denote the average complexity of Quicksort. Show that:

$$A(n) = (n - 1) + \frac{1}{n} \sum_{i=1}^n [A(i - 1) + A(n - i)]$$

2.b [6 marks] Reduce the above equation to:

$$\frac{A(n)}{n + 1} = \frac{A(n - 1)}{n} + \frac{2n - 2}{n(n + 1)}$$

2.c [6 marks] Solve the recurrence in part (b). Assume $\log_e(2) = 0.715$.

Question 3 *Marks 14*

3.a [6 marks] Consider a modified Quicksort algorithm that chooses the median, the middle value of an ordered list, to split a list into two parts. The new method first finds the median; then swaps the median with the first value of the array; and executes the Splitlist procedure as discussed in class.

Assuming that $3n$ steps are needed to find the median of n values, write a recurrence relation for the worst case complexity of the above algorithm.

3.b [8 marks] Write a recurrence relation for Mergesort given that the list to be sorted is already in increasing order. (Hint: The number of comparisons for merging sorted sub-lists will be different from the worst case.)

Solve this recurrence and give an exact expression for the complexity given a list with N numbers.

Question 4 *Marks 8*

You have 26 coins that are all supposed to be gold coins of the same weight, except one coin that is fake and weighs less. You are given a balance scale on which any number of coins can be put on either side, and the scale will tell you if the two sides weigh the same or if either side is lighter. Outline an efficient algorithm for finding the fake coin using atmost 3 weighings.

Question	Mark	Out Of
1	_____	10
2	_____	18
3	_____	14
4	_____	8
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Total	_____	50