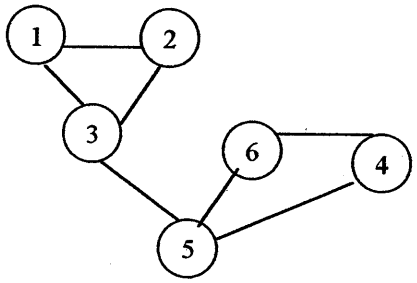
Final Exam: CMPUT 204
Section: B3, Friday, April 20th, 2001
Closed Book --- Only Pen, Pencil allowed
Time: 2hrs.; Max. mark: 100

1. (a) [5] Given a connected, undirected graph, what type of edges (among tree, back, and cross) are possible during Breadth First Search (BFS)?

(b) [5] Given a connected, DIRECTED graph instead, how will your answer to part (a) change?

(c) [10] Show stages of the DFS tree and edge stack while finding Bi-connected components of the following graph starting at vertex # 3. (Show the adjacency list of the graph to start with.)



(d) [5] Would the bi-connected component algorithm work properly if for each vertex v back[v] were initialized to DFS# of parent of v when it is first visited? If so, explain why; if not, give a counterexample.

(e) [5] Would the bi-connected component algorithm work properly if for each vertex v back[v] were initialized to $+\infty$?
If so, explain why; if not, give a counterexample.

(f) [5] Given a connected graph G, an edge e whose removal disconnects the graph is called a bridge. How can you modify the test to detect bi-connected components, to detect bridges instead?

2. (a) [10] Consider 9 elements stored in an array S using the Dynamic Equivalence relations on sets:

S

5	9	1	7	-6	-3	6	9	5

Show the steps performed by UNION(4,8) on the above sets, considering Weighted Union and Path Compaction. Also show the values stored in array S after the UNION(4,8) operation is complete.

(b) [5] Suppose you are given a graph with n vertices and $2n\sqrt{n}$ edges. Which of the MST algorithms discussed in class would you use to find a MST to minimize time complexity. Clearly justify your answer, no marks for guessing.

3. [20] Suppose the dimensions of 4 matrices A, B, C, D are 9 x 2, 2 x 6, 6 x 1, and 1 x 4 respectively. Following the dynamic programming algorithm discussed in class, determine the best order in which to multiply the matrices; show the steps in the dynamic programming algorithm used to derive the result, NO MARKS will be given for a trial & error solution.

What is the minimum number of scalar multiplications needed to compute
A x B x C x D.

4. (a) [5] You are given that CLIQUE is NP-complete.
What are the steps needed to show that another problem B, say, is NP-complete using the result that CLIQUE is NP-complete.

(b) [10] Consider the following sub-expression in SAT:

a AND (a v b) AND (a v b v c v d v e) AND ... (*)

What is an equivalent 3-SAT sub-expression which computes to TRUE if and only if the sub-expression in (*) is TRUE. Show the steps taken in obtaining the 3-SAT sub-expression.

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5. [15] You are given an array X of distinct positive integers with N values in it.
Give an efficient algorithm, with logarithmic complexity, to answer the question:
"Is there an index I such that X[I] = (I+3)?"
Justify why your algorithm will always work.