

CMPUT 204 Section B3 *Culberson*
MIDTERM Feb. 17, 2000
CLOSED BOOK. NO Notes or Calculators.
Time 70 minutes.
Answer all questions in the space provided.
Do scratch work on page backs.

Last Name: _____

First Name: _____

Make sure your name and ID is on the top of each internal page



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PAGES: 6

Question 1 Marks 6 Which of the following are true or false. BRIEFLY JUSTIFY YOUR ANSWERS.

1.a If $f(n) \in O(n^3)$ and $f(n) + g(n) \in \Theta(n^3)$ then $g(n) \in \Theta(n^3)$.

1.b $2^n \in O(3^n)$

1.c For sorting an array $A[1..n]$, Quicksort is always faster than Bubblesort if n is large. (Assume the version of Bubblesort used on the first assignment, and a version of Quicksort that picks its pivot at random).

Question 2 Marks 5 Let $T(1) = 3$ and $T(n) = 4T(n/3) + n$.

2.a Determine $T(n)$ for $n = 1, 3$ and 9 .

2.b Solve the recurrence exactly for n a power of 3, and then state your result using Θ notation.

Question 3 Marks 5 For the following algorithm, write a recurrence describing the runtime. Indicate clearly how the terms of your recurrence relate to the program. DO NOT try to solve this recurrence. Do not try to figure out what the algorithm is doing.

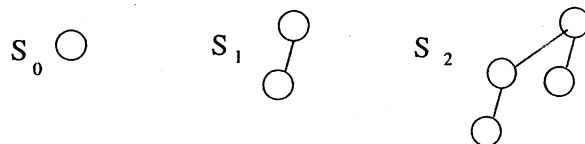
```
Proc DoSomething(int A[x..y]) {  
  
    if (y-x) < 3 then {  
        A[x] = 2*A[x];  
        return  
    }  
  
    nhalf = (y+x)/2;  
    DoSomething(int A[x..nhalf]);  
    DoSomething(int A[nhalf+1..y]);  
  
    MergeSort(A[x..y]);  
  
    DoSomething(int A[nhalf+1..y]);  
}  
Initially called by DoSomething(A[1..n]);
```

Question 4 Marks 4 Let $T(1) = 17$ and $T(n) = bT(n/c) + n^x$ for $n > 1$. Give the simplest big- Θ expression for $T(n)$ if b, c, x are respectively (i) 7,3,3 (ii) 16,4,2

(i)

(ii)

Question 5 Marks 6 S_k trees are defined as follows: S_0 is a tree with one node. For $k > 0$ an S_k tree is obtained from two S_{k-1} trees by making the root of one tree a child of the root of the other. Here are examples:



5.a How many nodes are in an S_k tree? Justify your answer

5.b What is the height (longest path from root to leaf) of an S_k tree? Justify your answer.

5.c Suppose we build a tree on n nodes by first building a sufficiently large S_k tree (using the minimal possible k), and then deleting leaves in an arbitrary order until only n nodes remain. What is the maximum possible height of this tree as a function of n ? Justify your answer.

Question 6 Marks 4 Dr. Joe decided to write a recursive program to pick an element of an n element array randomly.

```
RandPick(A[x..y]){  
    if (x == y) return A[x];
```

```
u = Random(x,y) // returns a random integer from [x, ... ,y-1]
```

```
RandPick(A[x..u])  
}
```

```
Called by RandPick(A[1..n])
```

6.a (1 mark) How random *is* the output?

6.b (3 marks) Write a recurrence relation describing the *average* number of calls made to the **Random** function. Do not try to solve this recurrence.

6.c (Bonus 2 marks) Solve this recurrence or give a simple argument to estimate a good Big-O bound on the solution.

Question	Mark	Out Of
1	_____	6
2	_____	5
3	_____	5
4	_____	4
5	_____	6
6	_____	4
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Total	_____	30