CMPUT 204 Winter 2001: Section B2
Midterm 1
Monday, Feb. 4
Time: 50 minutes
Weight 20%
Total Points 40

Last name:
First name:
Unix ID:

- No books or notes are allowed.
- No calculators or other mechanical devices are allowed.
- A signature sheet will be circulated.
- This exam has 5 pages and 7 questions. You are responsible for checking that your exam booklet is complete.

Question 1 [3 points]

Consider the following program:

```
int foo(int n) {
  int res = 1;
  int half = n/2;
  for (i=0;i<half;i++) {
    res += foo(i);
  }
  for (i=half;i<n;i++) {
    res += i;
  }
  return res;
}
```

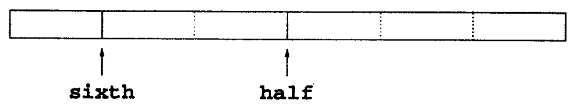
Write a recurrence relation expressing the number of additions that are done for the input n. Do not attempt to solve it.

Question 2 [2 points]
Give a big-Θ expression for $T(n)$ where

$$T(n) = \begin{cases} 64T(\frac{n}{4}) + n^2 & n > 0 \\ 1 & n = 0 \end{cases}$$

Question 3 [10 points]

Consider the following strange variant of binary search of a sorted array. The idea is to split the input array into 3 unequal pieces, and recursively continue the search in one of those pieces.




```
int Search(Element[] E, int low, int high, Key k) {
  int index = -1;
  if (low==high) {
    if (k == E[low].key) index = low;
  } else {
    int sixth = (5*low+high)/6;
    int half  = (  low+high)/2;
    if (k < E[sixth].key) {
      Search(E,low,sixth,k);
    } else if (k < E[half].key) {
      Search(E,sixth+1,half,k);
    } else {
      Search(E,half+1,high,k);
    }
  }
  return index;
}
```

Using the number of array (or subarray) elements as a measure of size, and using key comparisons as the basic operation, do the following:

3.a [2 pts]: State a recurrence relation for the worst case time.

3.b [2 pts]: State a recurrence relation for the best case time.

3.c [2 pts]: Solve the expression found for the best case. (You may assume n is an integral power of an appropriate base).

3.d [2 pts]: State a recurrence relation for the average time. Explicitly state any assumptions that you make.

3.e [2 pts]: State a big-Θ expression for the worst case space usage.

Question 4 [3 points]
Given two integers, m and n, of arbitrary size, what are the worst case costs for:

1. adding them
2. multiplying them
3. comparing them

Question 5 [8 points]

$$T(n) = \begin{cases} 2T(n/5) + n^2 & n > 1 \\ 1 & n = 1 \end{cases}$$

5.a [2 pts]: State the asymptotic complexity of of $T(n)$ (note: $\lg 5 \approx 2.3$)

5.b [6 pts]: Solve the recurrence exactly for n being a power of 5.

Question 6 [11 points]

6.a [3 pts]: Rank the following functions of n by their asymptotic (big-Θ) order. Indicate when functions are in the same complexity class.

$$(\ln n)^2 \qquad \ln(\ln n) \qquad n \ln n \qquad n^2 \ln n \qquad n \ln(n^2)$$

6.b [2 pts]: Give the mathematical definition of $f(n) \in O(g(n))$

6.c [3 pts]: If $f(n) \in \Theta(g(n))$ then which of the following are true:

1. $g(n) \in \Theta(f(n))$
2. $f(n) \in \Omega(g(n))$
3. $f(n) \in o(g(n))$

6.d [3 pts]: Give the simplest big-Θ expression for

$$(\pi n^3 + e^{n-1} + \log_7 n) \cdot (3n^2 + \ln(n^2))$$

Q	Mark	
1		3
2		2
3		10
4		3
5		8
6		11
7		3
Σ		40

Question 7 [3 points]

Solve the following recurrence relation exactly.

$$T(n) = \begin{cases} n + \sum_{i=0}^{n-1} T(i) & n > 0 \\ 0 & n = 0 \end{cases}$$