

CMPUT 204 Section B1 (Culberson)  
MIDTERM Mar. 12, 2001  
CLOSED BOOK. No Notes or Calculators.  
Time 50 minutes.  
Answer all questions in the space provided.  
Do scratch work on page backs

Last Name: \_\_\_\_\_

First Name: \_\_\_\_\_

Make sure your name and ID is on the top of each internal page.

**Question 1** Marks 4 Write a recurrence describing the number of swaps done by the following algorithm. Clearly indicate how your recurrence relates to the algorithm.

```
void forfun(int n, array A[])
{
    if (n <= 3) {
        for (i=1; i<= 3; i++)
            swap(A[i],A[3]);
    } else {
        for (i= 1; i<=n-1; i++) {
            swap(A[i], A[n-i]);
            forfun(i,A);
        }
    }
}
```

**Question 2** Marks 4 Analyze the runtime of the following algorithm giving your answer using the big- $\Theta$  notation. You may assume that initially it is called with  $n = 3^k$  for some integer  $k$ .

```
int morefun(int n)
{
    if (n==1) return 1;
    else {
        x = 0;
        for (i=1; i <= n*n; i++) {
            x = x + i;
        }

        x = x + morefun(n/3);
        x = x + morefun(n/3);
    }
}
```

**Question 3** *Marks 10* In class we discussed a binary heap in which each internal node in an implicit tree structure had two children (except possibly the last). We consider here a variation called a *ternary heap*. As the name suggests, these differ in that each internal node in a ternary heap has three children. Similar to the implementation of a binary heap a ternary heap is also stored in an array. The tree structure is implicit.

You may wish to draw a depth three ternary tree to assist in answering the following.

3.a Consider a node at index  $i$ , where  $i > 1$ .

(i) What are the indices of the three children of  $i$ ?

(ii) What is the index of the parent of  $i$ ?

3.b What is the maximum number of nodes at level  $k$  of a ternary tree?

3.c What is the maximum total number of nodes in a ternary tree of depth  $k$ ?

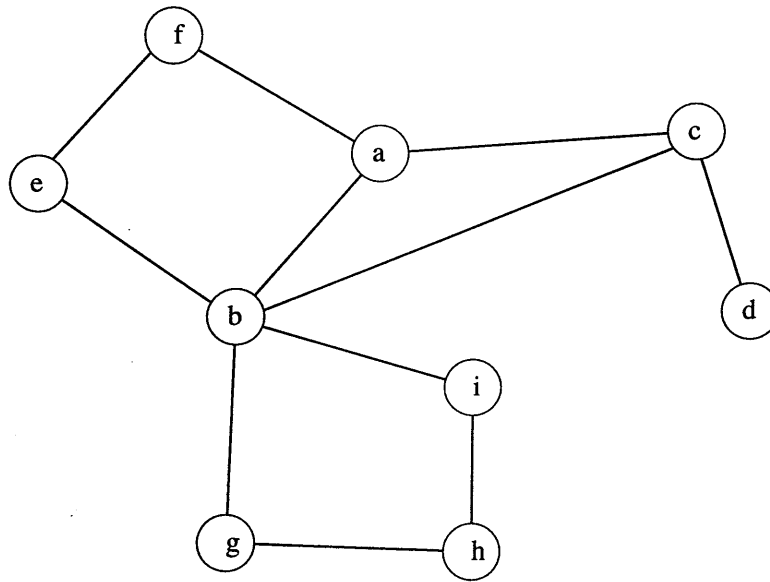
3.d Outline an algorithm (i.e. a high level description) for deleting the maximum value from a max ternary heap. Using this outline, how many comparisons would be necessary to delete the maximum from a ternary heap of depth  $k$  in the worst case?

**Question 4** *Marks 10* Let  $S = \{1, 2, \dots, 9\}$  and assume that wUnion and cFind are used. (If the sizes of the trees rooted at  $t$  and  $u$  are equal,  $\text{union}(t, u)$  makes  $u$  the root of the new tree.) Draw the trees after the last union and after each find in the following program.

```
union(1,2)
union(3,4)
union(2,4)
union(6,7)
union(8,9)
union(7,9)
union(4,9)
find(1)
find(6)
find(3)
```

**Question 5** Marks 12 Use the efficient bicomponent algorithm to find the bicomponents of the following graph. Start with vertex 'a' and assume the adjacency list of each vertex is in alphabetical order.

1. Draw the DFS tree clearly showing tree edges and back edges.
2. For each vertex show the discover time (i.e. the depth first number)
3. For each vertex show the final back number generated by the algorithm.
4. List the articulation points and the edges of each bicomponent.



Question	Mark	Out Of
1	_____	4
2	_____	4
3	_____	10
4	_____	10
5	_____	12
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Total	_____	40