CMPUT 204 Section B1 (Culberson)
MIDTERM Feb. 5, 2001
CLOSED BOOK. NO Notes or Calculators.
Time 50 minutes.
Answer all questions in the space provided.
Do scratch work on page backs.

Last Name:_____

First Name: _____

Make sure your name and ID is on the top of each internal page

Question 1 *Marks 7*

1.a [2 marks] Give the mathematical definition of big-O.

1.b [1 mark] Give the simplest big-Θ expression for $n^2 + 2^n + 3^n + n^3 \ln n$

1.c [1 mark] Which of the following are in $O(n^3)$:
$n^3, \ n^2 \log n, \ n^3 \log n, \ n(\log n)^3$

1.d [2 marks] Put the following in increasing order of complexity. Indicate any of equal complexity.
$2^n, \ n^2, \ n^2 \log n, \ n(\log n)^2, \ n^{0.001}, \ (\log n)^{10}$

1.e [1 mark] Suppose you are asked to write a program to multiply two integers of arbitrary size. How would you measure the size of the input? What operation would you count to estimate the runtime?

Question 2 *Marks 4* Let $T(1) = 1$ and $T(n) = bT(n/c) + n^x$ for $n > 1$. Give the simplest big-Θ expression for $T(n)$ if b, c, x are respectively (i) 10,3,3 (ii) 9,3,2

    (i)

    (ii)

Question 3 *Marks 5* For the following algorithm, write a recurrence describing the the number of swaps. Indicate clearly how the terms of your recurrence relate to the program. DO NOT try to solve this recurrence. Do not try to figure out what the algorithm is doing.

```
int Card[1..MAX];
Proc Mixup(int x,y) {
    if (x == y)
    {
        swap(Card[1],Card[x]);
    }
    else
    {
        n = y-x;
        for (i=1; i<n; i++)
        {
            swap(Card[x],Card[x+i]);
            Mixup(x,i);
            Mixup(i,y);
        }
    }
}
Initially called by Mixup(1,n);
```

Question 4 *Marks 4* The Zeta Company has to test a certain structure for stability deep in the ocean. The structure is constructed by piecing together 1 or more units. If there are too many units in the structure it will collapse. The Zeta Company has no idea how many units will lead to collapse, so they need to do tests. Each test costs $100000, since they must take the structure out to sea, sink it into deep water, and then retrieve it to see if it failed.

4.a Outline an algorithm to efficiently determine how many units will cause collapse.

4.b If the maximum safe structure consists of n units, how much will your method cost the Zeta Company to learn the value of n?

Question 5 *Marks 5* Here is an algorithm for Binary Search. It is to return the location of X in the sorted array **A** if it is in **A**, else return 0. You may assume that all elements in **A** are distinct.

```
int A[1..n];
BS(int first, last,X])
   while ( first < last) {
      mid = floor((first+last)/2);
      if ( X <= A[mid] )
         last = mid;
      else
         first = mid+1;
   }
   if A[first] == X return first;
   else              return 0;
}
```

5.a [3 marks] The algorithm makes decisions based on comparisons $A_i : X$ where $1 \leq i \leq n$. Draw the binary decision tree for this algorithm on a five element array $A_1, \ldots, A_5$. Each node of the tree should indicate the comparison made, and the values of `first` and `last` (use $f =$ and $\ell =$). The edges should be labeled with the decision outcome. The leaves of the tree should indicate the values returned.

5.b [2 marks] Give an argument that binary search is optimal over all binary decision tree search algorithms.

Question	Mark	Out Of
1	——	7
2	——	4
3	——	5
4	——	4
5	——	5
Total	——	25