

Friday, 18th February 2000.

CLOSED BOOK.

Questions to be answered in Examination Booklets.

Do All Questions. Time: 50 minutes. Total Marks 50.

1. (10 marks) Which of the following are true? Indicate briefly why or why not. (Assume $f(n)$ and $g(n)$ are positive non-decreasing functions.)

(a) if $f(n) \in O(n^2)$ and $g(n) \in O(n^2)$ then $\frac{f(n)}{g(n)} \in O(1)$.

(b) if $f(n) \in \Theta(n^3)$ and $g(n) \in \Theta(n)$ then $\frac{f(n)}{g(n)} \in \Theta(n^2)$.

(c) $n^\alpha \in \Theta(\log n)$ for some $\alpha > 0$.

(d) $\sum_{i=2}^{n-1} n^2 * i \in \Theta(n^4)$.

(e) if $f(n) \in o(n^2)$ and $g(n) \in \Theta(n^2)$ then $2^{f(n)} \in O(2^{g(n)})$.

2. (5 + 5 = 10 marks) Consider sorting a list of n distinct numbers using Quicksort.

(a) Let $A(n)$ denote the average complexity of Quicksort. Show that:

$$A(n) = (n - 1) + \frac{2}{n} \sum_{i=2}^{n-1} A(i)$$

(b) Reduce the above equation to a form that has no summation.

3. (a) (5 marks) Write two recurrence relations for (i) the number of *additions/subtractions*, and (ii) the number of *multiplications/divisions* performed by the following function. DO NOT SOLVE THE RECURRENCES.

function Weird(n : integer):integer;

begin

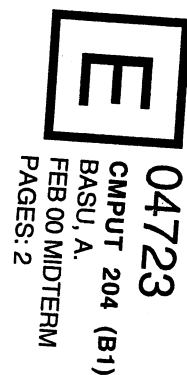
 if ($n \leq 3$) then Weird := 1

 else begin

 Weird := Weird($n-5$) + 2*Weird($n-2$) + Weird($n/2$);

 end;

end;



- (b) (5 + 5 = 10 marks) Write a recurrence relation for the number of key comparisons done by Mergesort if the n keys to be sorted are already in correct order when the sort begins. (Assume n is a power of 2.)

Give an exact solution of this recurrence.

4. (a) (10 marks) You are given 9 nodes $S = 1, 2, \dots, 9$, all in different sets (independent trees) to begin with. Consider the following sequence of Unions and Merges:

Union(6, 7), Union(8, 9), Merge(6, 8), Union(5, 4), Union(2, 3), Merge(2, 1), Union(3, 4), Union(9, 3).

[Assume Union(i, j) makes i point to j , if subtrees rooted at i & j have the same number of nodes.]

Show how the WEIGHTED Union builds the forest of trees for the above sequence.

Use the 1-D array data structure shown in class.

- (b) (5 marks) Show how the following FIND's (using Path Compression) modify the structure resulting at the end of part (a).

Find(6), Find(5).