Computing Science 204 Midterm Exam A. Basu B 1

Friday, 18th February 2000.

CLOSED BOOK.

Questions to be answered in Examination Booklets.

Do All Questions. Time: 50 minutes. Total Marks 50.

1. (10 marks) Which of the following are true? Indicate briefly why or why not. (Assume $f(n)$ and $g(n)$ are positive non-decreasing functions.)

   (a) if $f(n) \in O(n^2)$ and $g(n) \in O(n^2)$ then $\frac{f(n)}{g(n)} \in O(1)$.

   (b) if $f(n) \in \Theta(n^3)$ and $g(n) \in \Theta(n)$ then $\frac{f(n)}{g(n)} \in \Theta(n^2)$.

   (c) $n^\alpha \in \Theta(\ log\ n)$ for some $\alpha > 0$.

   (d) $\sum_{i=2}^{n-1} n^2 * i \in \Theta(n^4)$.

   (e) if $f(n) \in o(n^2)$ and $g(n) \in \Theta(n^2)$ then $2^{f(n)} \in O(2^{g(n)})$.

2. (5 + 5 = 10 marks) Consider sorting a list of n distinct numbers using Quicksort.

   (a) Let $A(n)$ denote the average complexity of Quicksort. Show that:

   $$A(n) = (n-1) + \frac{2}{n}\sum_{i=2}^{n-1} A(i)$$

   (b) Reduce the above equation to a form that has no summation.

3. (a) (5 marks) Write two recurrence relations for (i) the number of *additions/subtractions*, and (ii) the number of *multiplications/divisions* performed by the following function. DO NOT SOLVE THE RECURRENCES.

```
function Weird(n : integer):integer;
begin
    if ( n ≤ 3) then Weird := 1
    else begin
        Weird := Weird(n-5) + 2*Weird(n-2) + Weird(n/2);
    end;
end;
```

(b) (5 + 5 = 10 marks) Write a recurrence relation for the number of key comparisons done by Mergesort if the n keys to be sorted are already in correct order when the sort begins. (Assume n is a power of 2.)

Give an exact solution of this recurrence.

4. (a) (10 marks) You are given 9 nodes $S = 1, 2, ..., 9$, all in different sets (independent trees) to begin with. Consider the following sequence of Unions and Merges:

Union(6, 7), Union(8, 9), Merge(6, 8), Union(5, 4), Union(2, 3), Merge(2, 1), Union(3, 4), Union(9, 3).

[Assume Union(i, j) makes i point to j, if subtrees rooted at i & j have the same number of nodes.]

Show how the WEIGHTED Union builds the forest of trees for the above sequence.

Use the 1-D array data structure shown in class.

(b) (5 marks) Show how the following FIND's (using Path Compression) modify the structure resulting at the end of part (a).

Find(6), Find(5).