Final Exam: CMPUT 204 A. Basu B1
Section: B1, Friday April 26th, 2000
Full marks: 100
Closed book: only pen, pencil & eraser allowed
Time: 2hrs.

1. (a) [10] You have 81 coins that are all supposed to be gold coins of the same weight, except one coin that is fake and weighs less. Your are given a balance scale on which any number of coins can be put on either side, and the scale will tell you if the two sides weigh the same or which side is lighter. Outline an efficient algorithm for finding the fake coin in at most 4 weighing.

   (b) [10] Solve the following recurrence relation, given $T(2) = 1$ and $n = 2^{2^k}$ for some positive integer k.

   $$T(n) = 2\ T(\sqrt{n}) + c \qquad \text{(c is a positive constant)}$$

2. (a) [12] Which type of edges (tree, back, forward, cross) are possible for the following:
   (i) DFS of an undirected graph.
   (ii) BFS of an undirected graph.
   (iii) DFS of a directed graph.
   (iv) BFS of a directed graph.

   Justify your answers with a few lines of explanations as to why some types of edges may or may not occur.

   (b) [8] Consider the problem of finding the length of a shortest cycle in an undirected graph. Show why the following algorithm does not always work.

   When a back edge, say vw, is encountered during a DFS, it forms a cycle with the tree edges from w to v. The length of this cycle is depth[v] - depth[w] + 1, where depth[x] is the depth of vertex x in the DFS tree. The proposed algorithm works as follows: Do a DFS keeping track of the depth of each vertex in the DFS tree. Each time a back edge is encountered, compute the cycle length and save it if it is smaller than the shortest one previously seen.

3. (a) [8] A bi-partite graph is one whose vertices can be partitioned into two subsets such that there is no edge between any two vertices in the same subset. Write an algorithm using DFS to determine if an undirected graph is bi-partite.

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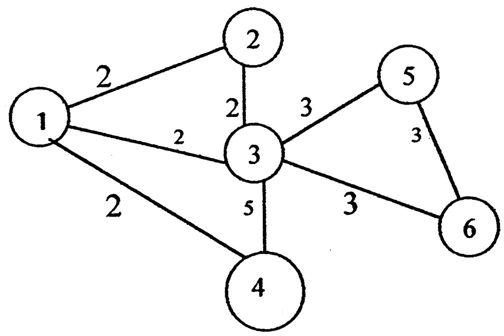
(b) [4] Consider the following graph. How many different Minimum Spanning Trees does this graph have? Justify your answer in a few sentences.



(c) [8] Consider a bi-partite graph G with n vertices, in which vertices 1 & 2 are in one set and the remaining vertices are in the second set. For this graph there is an edge between two vertices if and only if they are not in the same set. Given that all edges have the same weight, how many different Minimum Spanning Trees does G have? Justify your answer.

4. [20] Suppose the dimensions of 4 matrices A, B, C, D are 18 x 2, 2 x 16, 16 x 30, and 30 x 4 respectively. Following the dynamic programming algorithm discussed in class, determine the best order in which to multiply the matrices; show the steps in the dynamic programming algorithm used to derive the result, NO MARKS will be given for a trial & error solution.

What is the minimum number of scalar multiplications needed to compute A x B x C x D.

5. (a) [6] You are given that SAT is NP-complete.
   What are the steps needed to show that another problem B, say, is NP-complete Using the resilt that SAT is NP-complete.

   (b) [4] Is the Travelling Salesman problem NP-complete? If so, why? If not, why not?

   (c) [10] Consider the following sub-expression in SAT:

   (p v q v r v s v t) AND (u v q) AND ... (*)

   What is an equivalent 3-SAT sub-expression which computes to TRUE if and only if the sub-expression in (*) is TRUE. Show the steps taken in obtaining the 3-SAT sub-expression.