Sample all Sections

Instructor: Ryan Hayward

CS 204 Midterm November 8 2000 Version A

Write all answers in the space provided.

Time: 45 minutes. Total Marks: 20

1. (3 marks) For the following recurrence relations, use the master theorem to give the simplest Θ complexity of $R(n)$, or explain why the theorem does not apply.

   (a) $R(n) = 9R(n/2) + 2n^4 + 3n^2$ for $n \geq 2$, $R(1) = 1$.

   (b) $R(n) = 8R(n/2) + 7n^3$ for $n \geq 2$, $R(1) = 1$.

2. (3 marks) For $n \geq 2$, an n-by-n matrix multiplication algorithm performs, on $n/2$-by-$n/2$ submatrices, 5 recursive matrix multiplications and 7 non-recursive matrix additions; for $n = 1$ the algorithm performs one scalar multiplication.

   Give a recurrence relation describing exactly $T(n)$, the total number of scalar operations (addition plus multiplication) performed by this algorithm.

3. (4 marks) Explain in detail why $A(n)$, the average number of key comparisons made by a quicksort of n keys, satisfies

$$Q(n) = n - 1 + \frac{2}{n}\sum_{j=0}^{n-1} Q(j).$$

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1: 5 6
2: 7 8
3: 5 6 7
4: 8
5: 1 3 6
6: 1 3 5
7: 2 3 8
8: 2 4 7
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4. (6 marks) (a) Draw the graph represented above (beside its representation).

   (b) Starting from vertex 4 (i) list the vertices in BFS order

   (ii) draw the DFS recursion tree (label each tree node with the DFS call made).

   (c) Assume that for the above graph each edge $\{x, y\}$ with $x < y$ has weight $10x + y$. List the edges of an MST in the order in which they would be found by Kruskal's algorithm.

5. (4 marks) (a) Describe briefly an (efficient) algorithm which reports whether an input graph is a forest.

   (b) Give (and justify) the running time of your algorithm assuming the graph is represented by an adjacency matrix.